# MATH 101 Mathematics for Social Sciences I


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# Outline

**Lecture 5**

Chapter 3 Lines, Parabolas, and Systems

- Sec. 3.1 Lines

## Definition

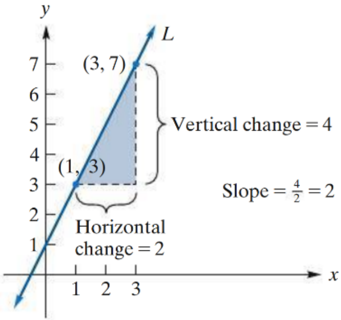
Let $(x_1, y_1)$ and $(x_2, y_2)$ be two different points on a nonvertical line. The **slope** of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{\text{vertical change}}{\text{horizontal change}}\right)$$

**Example:**

Consider the line $L$ passing through the points $(1, 3)$ and $(3, 7)$. Then $L$ has slope

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{7-3}{3-1} = 2.$$



## Vertical and horizontal lines

- A vertical line does not have a slope, since any two points on it has $x_1 = x_2$.
- A horizontal line has slope zero , since any two points on it has $y_1 = y_2$.

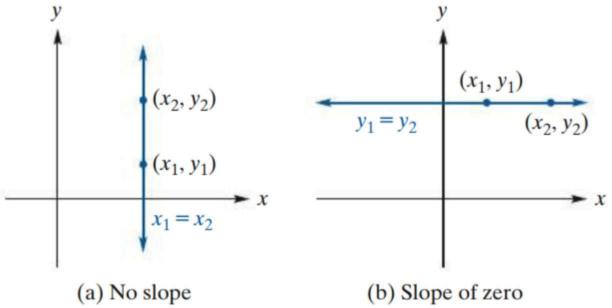



**FIGURE 3.3** Vertical and horizontal lines.

## Positive and negative slopes

- A line with positive slope rises from left to right.
- A line with negative slope falls from left to right.
- The closer the slope is to 0, the more nearly horizontal is the line.
- The greater the absolute value of the slope, the more nearly vertical is the line.

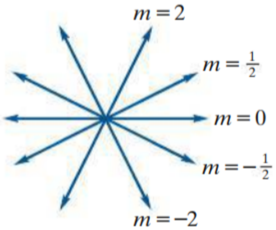



**FIGURE 3.5** Slopes of lines.

## Question

How to find a line's equation, given a point on the line and the slope?

Suppose the line $L$ has slope $m$ and passes through the point $(x_1, y_1)$. If $(x, y)$ is any other point on $L$, then we have

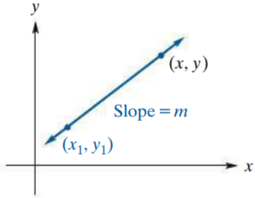



**FIGURE 3.6** Line through $(x_1, y_1)$ with slope $m$.

$$\frac{y - y_1}{x - x_1} = m$$

$$y - y_1 = m(x - x_1)$$

An equation for the line passing through $(x_1, y_1)$ with slope $m$ can be written in **point-slope form** as $y - y_1 = m(x - x_1)$.

**Example**

Find an equation of the line that has slope 2 and passes through $(1, -3)$.

**Solution:** Using the point-slope form with $m = 2$ and $(x_1, y_1) = (1, -3)$,

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= 2(x - 1) \\ y + 3 &= 2x - 2 \\ y &= 2x - 5 \end{aligned}$$

## Example

Find an equation of the line passing through $(-3, 8)$ and $(4, -2)$.

**Solution:** In order to apply the point-slope form, we need the slope which we can find from the given points.

$$m = \frac{-2 - 8}{4 - (-3)} = -\frac{10}{7}.$$

Now, taking $m = -10/7$ and $(x_1, y_1) = (-3, 8)$, we get

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= -\frac{10}{7}(x - (-3)) \\ y &= -\frac{10}{7}x + \frac{26}{7} \end{aligned}$$

If we know the $y$-intercept $b$ of a given line, then the line passes through $(0, b)$, and from the point-slope form we have

$$y - b = m(x - 0) \Rightarrow y = mx + b.$$

An equation of the line with slope $m$ and $y$-intercept $b$ can be written in **slope-intercept form** as $y = mx + b$.

**Example**

Find an equation of the line with slope 3 and $y$-intercept $-4$.

**Solution:** Using the slope-intercept form with $m = 3$ and $b = -4$,

$$y = mx + b \Rightarrow y = 3x + (-4) \Rightarrow y = 3x - 4.$$

### Example

Find the slope and $y$-intercept of the line with equation $y = 5(3 - 2x)$.

**Solution:** The slope-intercept form of the line immediately tells us the slope and the $y$-intercept. We have

$$y = 5(3 - 2x) \Rightarrow y = 15 - 10x \Rightarrow y = -10x + 15.$$

So, the slope is $-10$ and the $y$-intercept is 15.

### General linear form

Every straight line is the graph of the form $Ax + By + C = 0$, where $A$, $B$, and $C$ are constants such that $A$ and $B$ are not both zero.

**Example**

Find a general linear form of the line given by $y = -\frac{2}{3}x + 4$.

**Solution:** Getting one side of the equation to 0, we get

$$\frac{2}{3}x + y - 4 = 0$$

which is a general linear form with $A = 2/3$, $B = 1$ and $C = -4$.
Note that there may be other general linear forms of the same line. For example,

$$2x + 3y - 12 = 0$$

is another general linear form when we multiply both sides by 3.

## Parallel lines

Two lines are **parallel** if and only if they have the same slope or are both vertical.

## Perpendicular lines

Two lines with slopes $m_1$ and $m_2$ are **perpendicular** to each other if and only if

$$m_1 = -\frac{1}{m_2}.$$

Moreover, any horizontal line and any vertical line are perpendicular to each other.

### Example

Find an equation for the line passing through $(3, -2)$

(a) parallel to the line $y = 3x + 1$,

(b) perpendicular to the line $y = 3x + 1$.

**Solution:**

(a) The slope of $y = 3x + 1$ is 3. So, the parallel line has slope $m = 3$. Using the point-slope form with $m = 3$ and $(x_1, y_1) = (3, -2)$, we get

$$y - (-2) = 3(x - 3) \Rightarrow y + 2 = 3x - 9 \Rightarrow y = 3x - 11.$$

(b) The slope of the perpendicular line must be $-\frac{1}{3}$.
Using the point-slope form with $m = -\frac{1}{3}$ and $(x_1, y_1) = (3, -2)$, we get

$$y - (-2) = -\frac{1}{3}(x - 3) \Rightarrow y + 2 = -\frac{1}{3}x + 1 \Rightarrow y = -\frac{1}{3}x - 1.$$