MATH 101 Mathematics for Social Sciences I

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Outline

Lecture 3

Chapter 2 Functions and Graphs

- Sec. 2.5 Graphs in rectangular coordinates

Rectangular coordinate system:

- To represent a **rectangular coordinate plane**, two real number lines, called **coordinate axes**, are constructed perpendicular to each other.
- Their point of intersection is the **origin** where both coordinate values are 0.
- The horizontal line is the x-**axis** and the vertical line is the y-**axis**.
- Any point P in the plane can be represented by its **coordinates** as an **ordered pair** of the form (a, b).

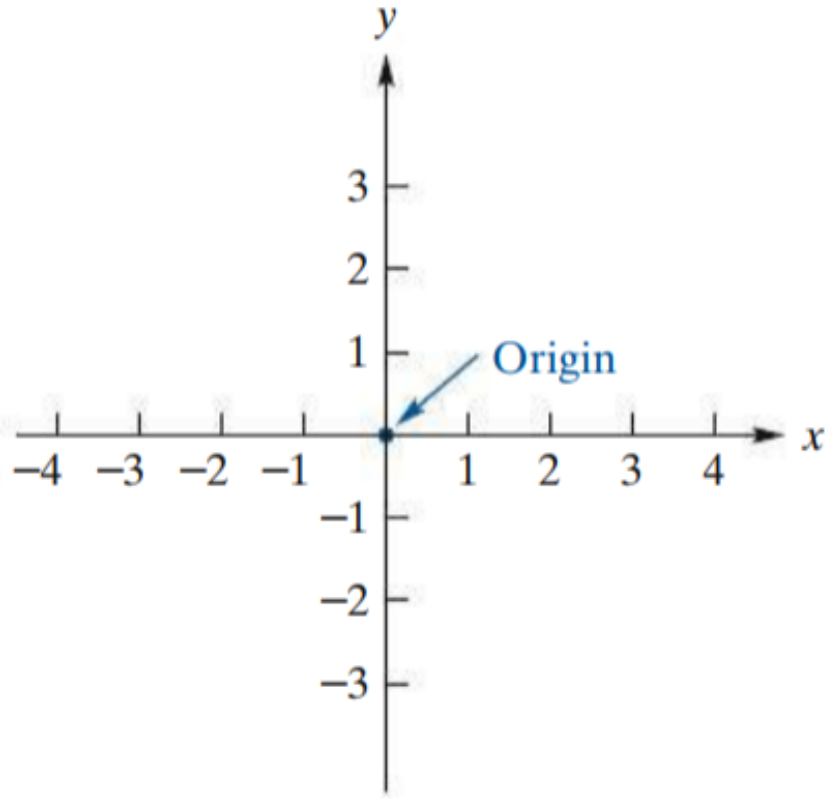


FIGURE 2.4 Coordinate axes.

Example

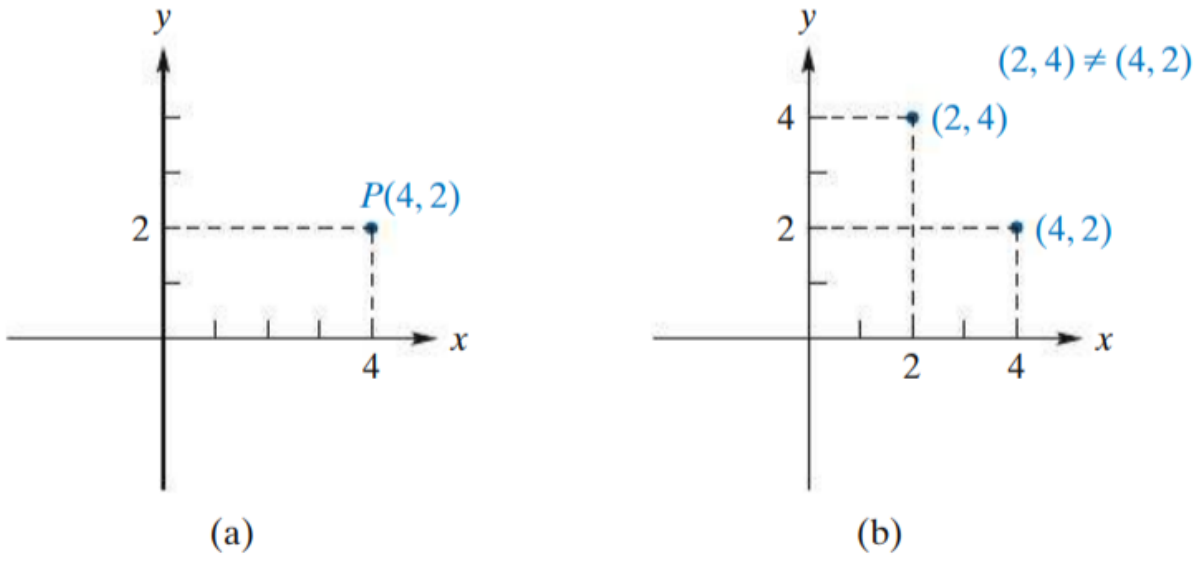


FIGURE 2.5 Rectangular coordinates.

There is a one-to-one correspondence between the points in the plane and all ordered pairs of real numbers!

The coordinate axes divide the plane into four regions called **quadrants**.

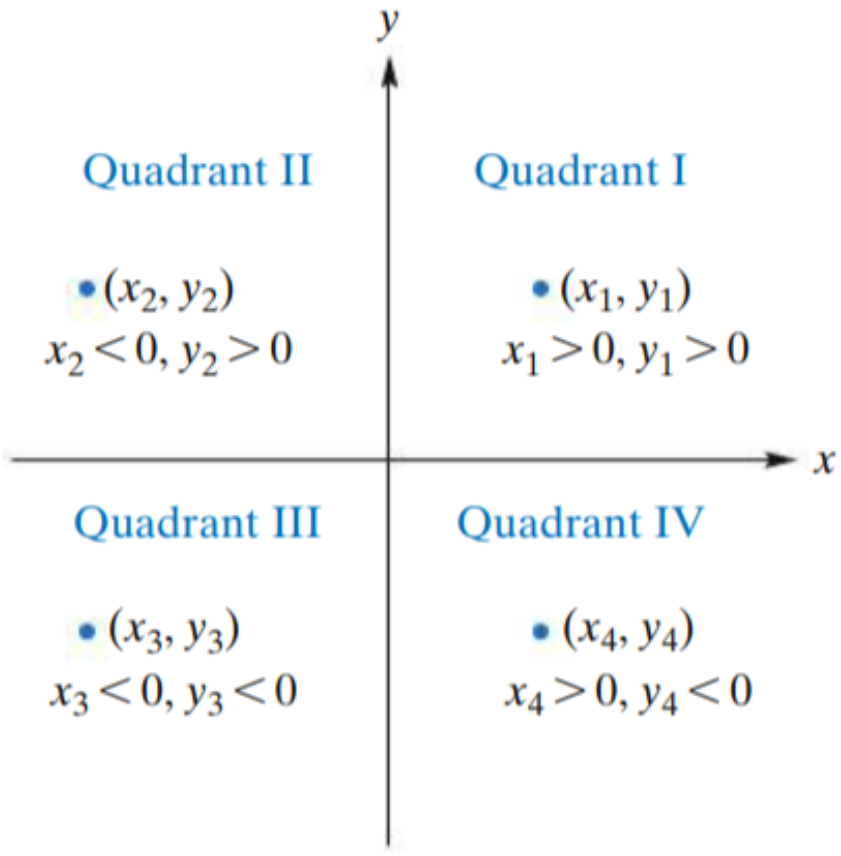


FIGURE 2.8 Quadrants.

Any equation in two variables can be represented geometrically in the rectangular coordinate system. Each function f gives us an equation

$$y = f(x)$$

and its **graph** consists of all points $(x, f(x))$, where x is in the domain of f.

Definition

An x-**intercept** of the graph of an equation in x and y is a point where the graph intersects the x-axis. A y-**intercept** is a point where the graph intersects the y-axis.

An x-intercept has its y-coordinate 0, and a y-intercept has its x-coordinate 0.

Example

Find the intercepts of $y = x^2 + 2x - 3$.

Solution: To find the y-intercept, we set $x = 0$ and solve for y:

$$y = (0)^2 + 2(0) - 3 = -3$$

So, the y-intercept is the point $(0, -3)$.
To find the x-intercepts, we set $y = 0$ and solve for x:

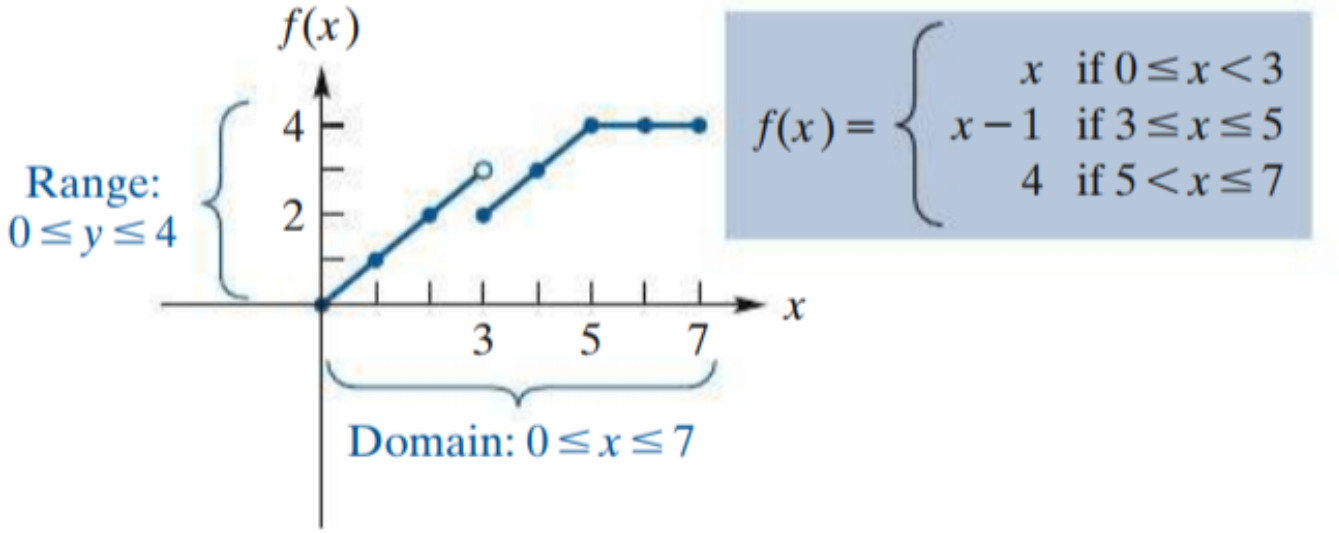
$$\begin{aligned} 0 &= x^2 + 2x - 3 \\ 0 &= (x+3)(x-1) \\ x &= -3,\ x = 1 \end{aligned}$$

Therefore, the x-intercepts are $(-3, 0)$ and $(1, 0)$.

Example

Graph the case-defined function $f(x) = \begin{cases} x, & 0 \le x < 3, \\ x-1, & 3 \le x \le 5, \\ 4, & 5 < x \le 7. \end{cases}$

Solution: The graph of f is given below. Note that a **hollow dot** at $(3, 3)$ means that that point is not included in the graph.



In general, the domain consists of all x-values that are included in the graph, and the range is all y-values that are included.

Remember that if f is a function, then every input x has at most one output $f(x)$. We can verify this geometrically as follows:

Vertical line test

If a vertical line L can be drawn that intersects a curve in at least two points, then the curve is **not** the graph of a function of x. When no such vertical line can be drawn, the curve **is** the graph of a function f.

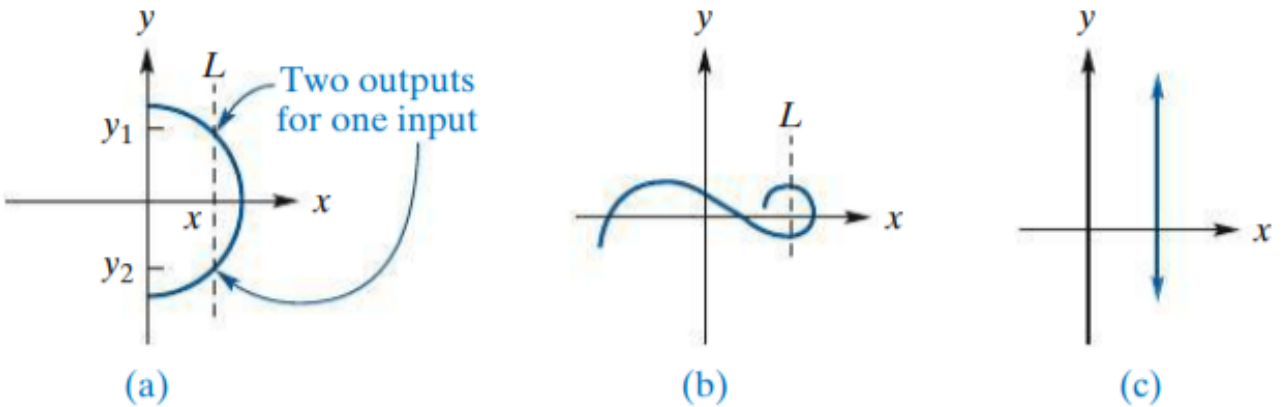


FIGURE 2.18 y is not a function of x.

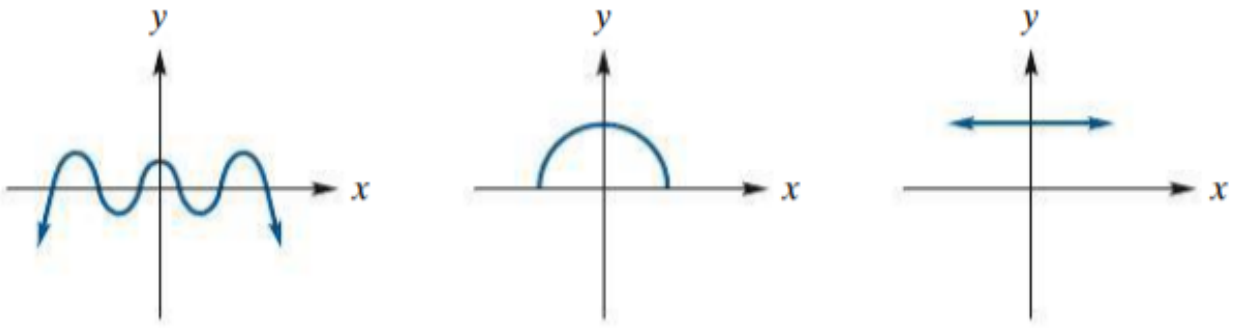


FIGURE 2.19 Functions of x.

Remember that if a function is one-to-one, then it should send distinct inputs to distint outputs. We can verify this geometrically as follows:

Horizontal line test

If a horizontal line L can be drawn that intersects the graph of a function in at least two points, then the function is **not** one-to-one. When no such horizontal line can be drawn, the function **is** one-to-one.

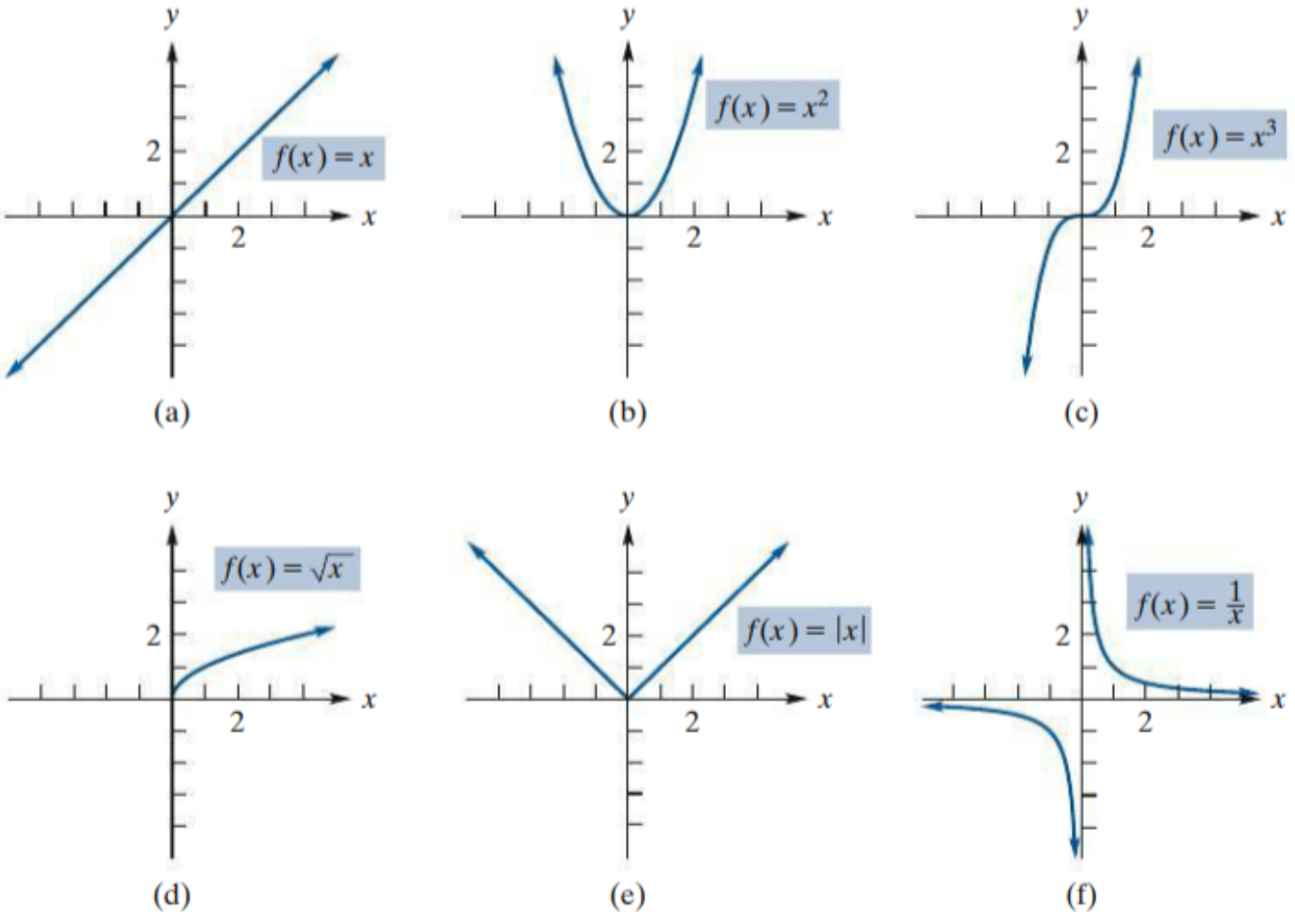


FIGURE 2.32 Six basic functions.