MATH 101 Mathematics for Social Sciences I

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Outline

Lecture 1

- Syllabus review
- Chapter 2 Functions and Graphs
  - Sec. 2.1 Functions
  - Sec. 2.2 Special functions

Definition

A **function** $f : X \to Y$ is a rule that assigns to each of certain elements x of the set X at most one element $f(x)$ of the set Y.

In this course, we will mainly consider functions where X and Y are both the set of all real numbers $(-\infty, \infty)$.

The subset of X consisting of all x for which $f(x)$ is defined is called the **domain** of f. Such x-values are called **input**s of f.

The set of all elements in Y of the form $f(x)$ for some x in X is called the **range** of f. Such values are called **output**s of f.

Example: Find the domain of each function.

a. $f(x) = \dfrac{x}{x^2 - x - 2}$

Since we cannot divide by zero, we cannot have the denominator

$$x^2 - x - 2 = 0 \Rightarrow (x+1)(x-2) = 0 \Rightarrow x = -1,\ x = 2.$$

Therefore, the domain of f is all real numbers except -1 and 2. We can write this as $(-\infty, \infty)\backslash\{-1, 2\}$.

b. $g(t) = \sqrt{2t - 1}$

The square root only outputs real values for non-negative inputs, so $\sqrt{2t-1}$ is a real number if $2t - 1 \geq 0 \Rightarrow t \geq 1/2$.
Therefore, the domain of g is the set of all real numbers greater than or equal to $1/2$, which is the interval $[1/2, \infty)$.

Inequality	Graph	Interval Notation
$a \le x \le b$		$[a,b]$
$a < x < b$		(a,b)
$a \le x < b$		$[a,b)$
$a < x \le b$		$(a,b]$
$x > a$		(a,∞)
$x \ge a$		$[a,\infty)$
$x < b$		$(-\infty,b)$
$x \le b$		$(-\infty,b]$

Equality of functions

Two functions $f, g : X \to Y$ are equal, denoted by $f = g$ whenever

1. The domain of f is equal to the domain of g as sets,
2. For every x in the domain of f and g, $f(x) = g(x)$.

Example

Determine which of the following functions are equal.

a. $f(x) = \dfrac{(x+2)(x-1)}{x-1}$

b. $g(x) = x + 2$

c. $h(x) = \begin{cases} x+2, & x \neq 1 \\ 0, & x = 1 \end{cases}$

d. $k(x) = \begin{cases} x+2, & x \neq 1 \\ 3, & x = 1 \end{cases}$

Answer: $g = k$

Constant functions

A function of the form $h(x) = c$, where c is a constant number, is called a **constant function**.

Example: Let $h : (-\infty, \infty) \to (-\infty, \infty)$ given by $h(x) = 2$. Then h is a constant function with domain $(-\infty, \infty)$ and range $\{2\}$.

Linear functions

A function of the form $f(x) = ax + b$, where a and b are constants and $a \neq 0$, is called a **linear function**.

Example: $g(x) = 4 - 2x$ is linear. Note that the domain of g is $(-\infty, \infty)$.

Polynomial functions

A function of the form

$$f(x) = c_n x^n + c_{n-1} x^{n-1} + \cdots + c_1 x + c_0$$

where n is a nonnegative integer and $c_n, c_{n-1}, \ldots, c_0$ are constants with $c_n \neq 0$ is called a **polynomial function**.

The number n is the **degree** of the polynomial, and c_n is the **leading coefficient**. The domain of any polynomial function is the set of all real numbers $(-\infty, \infty)$.

Examples:

a. $f(x) = 3x^2 - 8x + 9$ is a polynomial of degree 2 with leading coefficient 3. Polynomial functions of degree 2 are called **quadratic functions**.
b. $g(x) = \dfrac{2x}{3}$ has degree 1 with leading coefficient $\frac{2}{3}$. Note that all linear functions are polynomials with degree 1.
c. $r(x) = \dfrac{2}{x^3}$ is **not** a polynomial function since $r(x) = 2x^{-3}$ and the exponent -3 is not a nonnegative integer.
d. $s(x) = \sqrt{x}$ is **not** a polynomial, similarly because $s(x) = x^{1/2}$.

Rational functions

A quotient of polynomial functions is called a **rational function**, that is,

$$r(x) = \frac{p(x)}{q(x)},$$

where p and q are polynomials, and $q(x) \neq 0$.

Examples:

a. $f(x) = \dfrac{x^2 - 6x}{x + 5}$ is a rational function, since the numerator and the denominator are both polynomials.

b. $g(x) = 2x + 3$ is a rational function since $g(x) = \dfrac{2x + 3}{1}$.

Fact: Every polynomial function is also a rational function.

Absolute-value function

The function

$$|.|(x) = |x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0, \end{cases}$$

is called the **absolute-value function**.

The domain of the absolute-value function is all real numbers $(-\infty, \infty)$. Some function values are

$$|16| = 16, \quad \left|-\frac{4}{3}\right| = -\left(-\frac{4}{3}\right) = \frac{4}{3}, \quad |0| = 0.$$

Remark: The absolute-value function is an example of a **case-defined function**, where the rule for specifying it is given by rules for more than one disjoint cases.