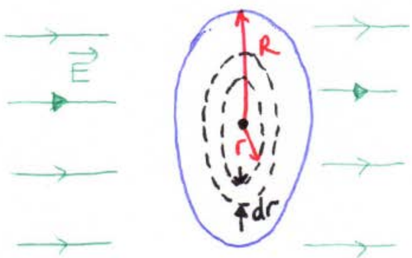
# 2014/2 ENGINEERING DEPARTMENTS PHYSICS 2

# RECITATION 2

# (Gauss's Law)

**1.** Consider an electric field in the constant direction is perpendicular to plane of a circle with radius **R**. The magnitude of electric field at a distance r from the center of the circle is $E_0\left[1-\dfrac{r}{R}\right]$. Determine the electric flux through the circle.




$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA = E_0\left(1-\frac{r}{R}\right)2\pi r\,dr$$

$$\Phi = \int E\,dA = \int E_0\left(1-\frac{r}{R}\right)2\pi r\,dr$$

$$\Phi = E_0\,2\pi\int_0^R\left(1-\frac{r}{R}\right)r\,dr$$

$$\Phi = E_0\,2\pi\left(\frac{r^2}{2}-\frac{r^3}{3R}\right)\Bigg|_0^R$$

$$\Phi = \pi E_0\frac{R^2}{3}$$

**2.** Consider a closed triangular box resting within a horizontal electric field of magnitude **E=7.80x10$^4$ (N/C)** as shown in **Figure 1**.Calculate the electric flux through

**a)** the vertical rectangular surface,

**b)** the slanted surface,

**c)** the entire surface of the box.

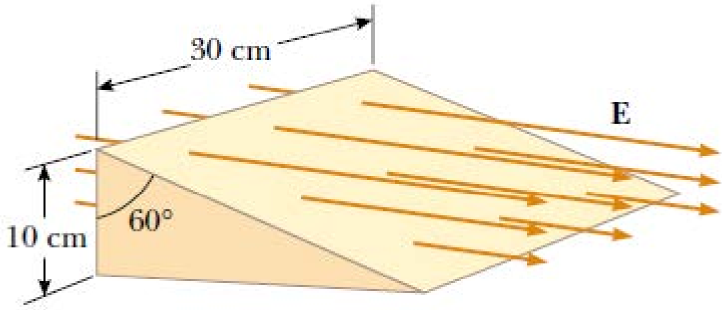
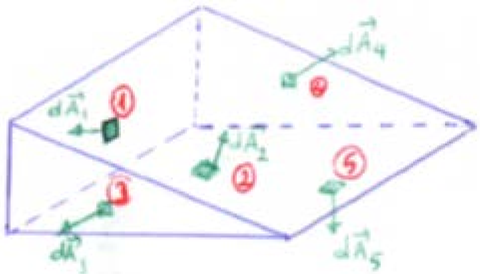
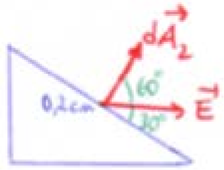



**Figure 1**




a) $\Phi_1 = EA_1 \cos\theta_1 = 7{,}8\cdot10^4\,(0{,}1\cdot0{,}3)\cos180^\circ = -2{,}34\ Nm^2/C$

b)




$\Phi_2 = EA_2\cos60^\circ = 7{,}8\cdot10^4\,(0{,}2\cdot0{,}3)\cos60^\circ$

$\Phi_2 = 2{,}34\ Nm^2/C$

c) The flux through the base 5, the front 3 and the back 4 surface of the box is zero. Because, the electric field vector is perpendicular to the surface.

$$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5$$

$$\Phi_{net} = -2{,}34 + 2{,}34 = 0\ Nm^2/C$$

**3.** A closed surface with dimensions **a=0.2 m**, **b=0.3 m** and **c=0.3 m** is located as in **Figure 2**. The left edge of the closed surface is located at position **x=a**. The electric field throughout the region is nonuniform and given by $E=(1+x^2)$ N/C, where **x** is in meters.

**a)** Calculate the net electric flux leaving the closed surface.

**b)** What net charge is enclosed by the surface?

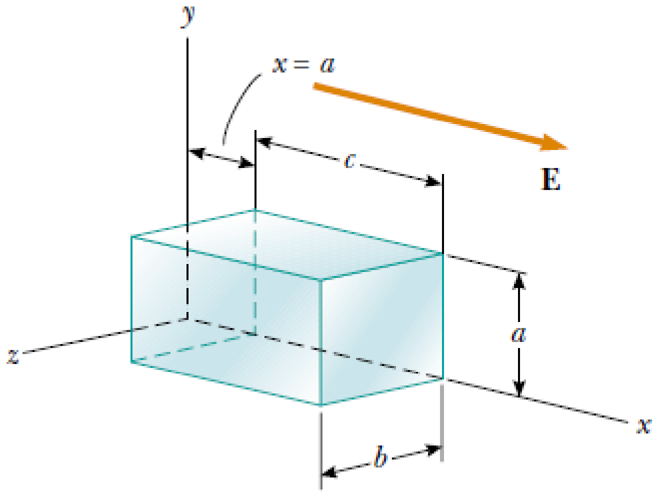
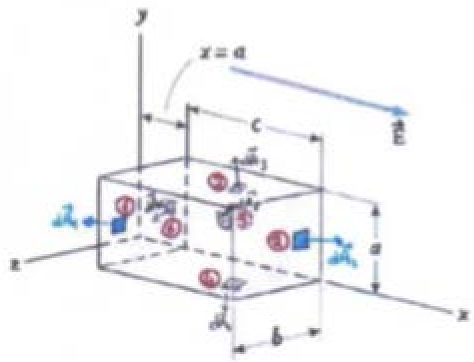



**Figure 2**




a) $\Phi_E = \oint \vec{E}\cdot d\vec{A}$

$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$

$\Phi_3 = \int_3 \vec{E}\cdot d\vec{A} = \int_3 E\,dA\cos 90^\circ = 0$

Same way,

$\Phi_4 = \Phi_5 = \Phi_6 = 0$

$\Phi_E = \Phi_1 + \Phi_2$

$\vec{E}_1 = (1+x^2)\hat{i}\Big|_{x=a} = (1+a^2)\hat{i}$ (N/C)

$\vec{E}_2 = (1+x^2)\hat{i}\Big|_{x=a+c} = \left[1+(a+c)^2\right]\hat{i}$ (N/C)

$\Phi_E = \int_1 \vec{E}_1\cdot d\vec{A}_1 + \int_2 \vec{E}_2\cdot d\vec{A}_2$

$\Phi_E = \int_1 (1+a^2)\hat{i}\cdot dA_1(-\hat{i}) + \int_2 \left[1+(a+c)^2\right]\hat{i}\cdot dA_2\,\hat{i}$

$$\Phi_E = -(1+a^2)\int_1 dA_1 + \left[1+(a+c)^2\right]\int_2 dA_2$$

$$\Phi_E = -(1+a^2)\,ab + \left[1+(a+c)^2\right]ab$$

$$\Phi_E = -ab - a^3b + ab + a^3b + 2a^3bc + abc = abc\,(2a+c)$$

$$\left.\begin{aligned} a &= 0{,}2\,m \\ b &= 0{,}3\,m \\ c &= 0{,}3\,m \end{aligned}\right\} \quad \Phi_E = 12{,}6\cdot 10^{-3}\ Nm^2/C$$

b) $\Phi_E = \dfrac{q_{net}}{\epsilon_0} \Rightarrow q_{net} = \epsilon_0\,\Phi_E$ $\qquad \epsilon_0 = 8{,}85\cdot 10^{-12}\ C^2/Nm^2$

$$q_{net} = 8{,}85\cdot 10^{-12}\cdot 12{,}6\cdot 10^{-3}$$

$$q_{net} = 1{,}12\cdot 10^{-13}\ C$$

**4.** Three infinite, nonconducting sheets of charge are parallel to each other, as shown in **Figure 3**. The sheets have a uniform surface charge density $\sigma_1 = +5(\mu C/m^2)$, $\sigma_2 = -10(\mu C/m^2)$ and $\sigma_3 = +15(\mu C/m^2)$, respectively.

Calculate the electric field at

**a)** I zone,

**b)** II zone,

**c)** III zone,

**d)** IV zone.

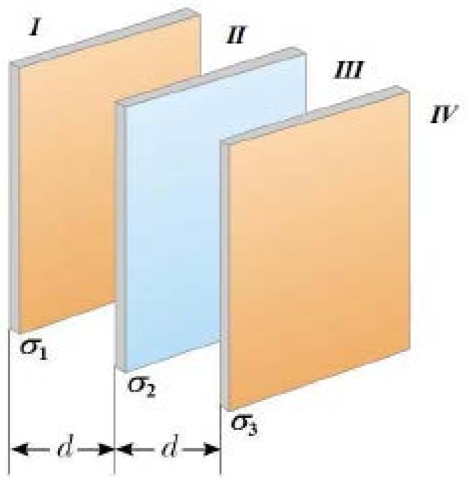
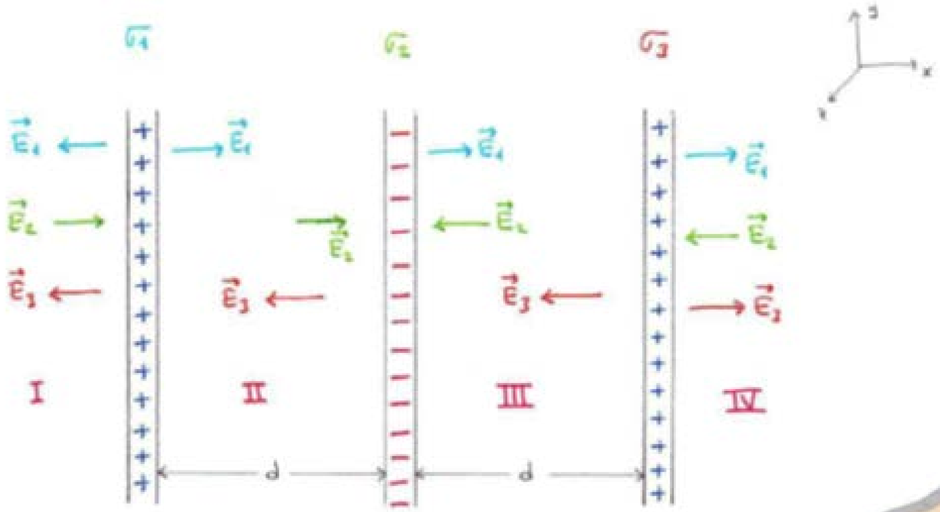


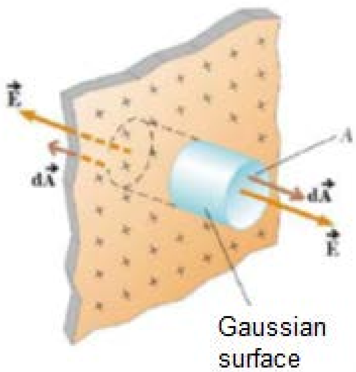

**Figure 3**




$$E_1 = \frac{\sigma_1}{2\epsilon_0} \qquad E_2 = \frac{\sigma_2}{2\epsilon_0} \qquad E_3 = \frac{\sigma_3}{2\epsilon_0}$$

$$E_1 = \frac{5.10^{-6}}{2.8,85.10^{-12}} \qquad E_2 = \frac{10.10^{-6}}{2.8,85.10^{-12}} \qquad E_3 = \frac{15.10^{-6}}{2.8,85.10^{-12}}$$

$$E_1 = 2,82.10^5 \ (N/c) \qquad E_2 = 5,65.10^5 \ (N/c) \qquad E_3 = 8,47.10^5 \ (N/c)$$




$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_i}{\epsilon_0}$$

$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

I zone ; $\vec{E}_{I} = E_1(-\hat{i}) + E_2(\hat{i}) + E_3(-\hat{i})$

$\vec{E}_{I} = (-2{,}82 + 5{,}65 - 8{,}47) . 10^5 \hat{i}$

$\boxed{\vec{E}_{I} = 5{,}64 . 10^5 (-\hat{i}) (N/C)}$

II zone ; $\vec{E}_{II} = E_1(\hat{i}) + E_2(\hat{i}) + E_3(-\hat{i})$

$\vec{E}_{II} = (2{,}82 + 5{,}65 - 8{,}47) . 10^5 \hat{i}$

$\boxed{\vec{E}_{II} = 0}$

III zone ; $\vec{E}_{III} = E_1(\hat{i}) + E_2(-\hat{i}) + E_3(-\hat{i})$

$\vec{E}_{III} = (2{,}82 - 5{,}65 - 8{,}47) . 10^5 \hat{i}$

$\boxed{\vec{E}_{III} = 11{,}30 . 10^5 (-\hat{i}) (N/C)}$

IV zone ; $\vec{E}_{IV} = E_1(\hat{i}) + E_2(-\hat{i}) + E_3(\hat{i})$

$\vec{E}_{IV} = (2{,}82 - 5{,}65 + 8{,}47) . 10^5 \hat{i}$

$\boxed{\vec{E}_{IV} = 5{,}64 . 10^5 (\hat{i}) (N/C)}$

**5.** A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire as in **Figure 4**. The wire has a charge per unit length of **+λ**, and the cylinder has a net charge per unit length of **+2λ**. From this information, use Gauss's law to find the electric field in the regions

**a)** r<a,

**b)** a<r<b,

**c)** r>b.

**d)** Determine the charge distribution of the cylindrical sheet.

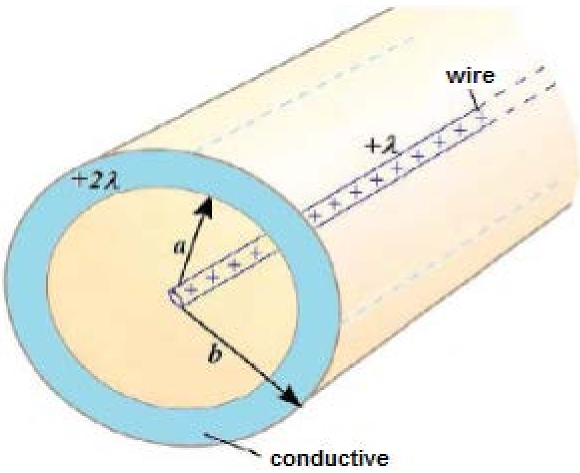
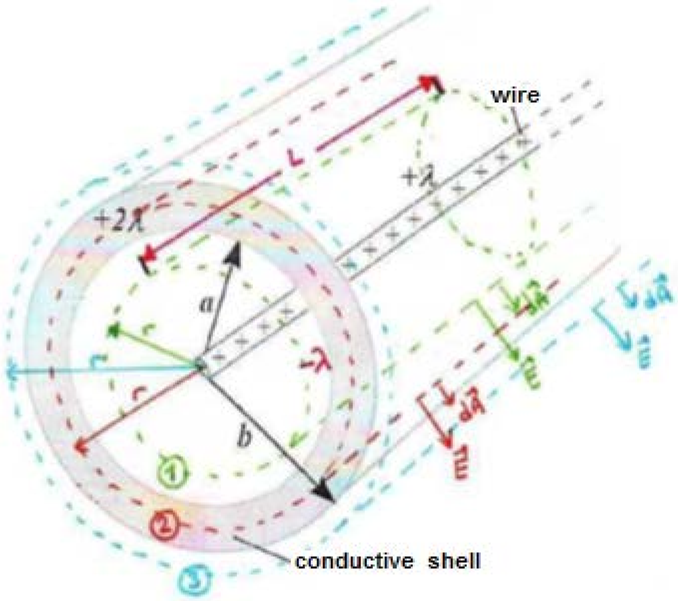



**Figure 4**




$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_i}{\epsilon_0}$$

a) $q_i = \lambda L$

$$E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

$$\boxed{E = 2k\frac{\lambda}{r}} \quad r<a$$

b) inside the conducting shell $E=0$

$$\boxed{E=0} \quad a<r<b$$

c) $$E(2\pi r L) = \frac{\lambda L + 2\lambda L}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{3\lambda}{r}$$

$$\boxed{E = 6k\frac{\lambda}{r}} \quad r > b$$

d) $$q_i = -\lambda L$$

Because, wire induces the inner surface of the cylinder

$$q_{cylinder} = q_i + q_{out}$$

$$\lambda_{cylinder} \cdot L = -\lambda L + q_{out}$$

$$2\lambda L + \lambda L = q_{out}$$

$$\boxed{q_{out} = 3\lambda L}$$

**6.** There is a +2Q point charge at the centre of an empty insulating sphere which carriers +Q total charge and has charge density, ρ.

**a)** Find the electric fields for R<r<2R and r>2R regions in terms of k, Q, r, and R.

**b)** If the sphere is conductor, calculate the electric fields for R<r<2R and r>2R regions.

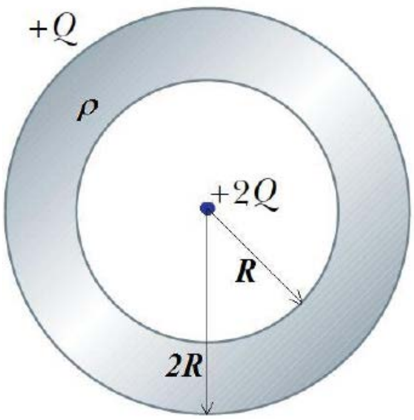



**Figure 5**

a) $\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

for $R<r<2R$ zone (1)

$$E(4\pi r^2) = \frac{q_{in}}{\epsilon_0} = \frac{2Q + q_{kure}}{\epsilon_0}$$

$$E(4\pi r^2) = \left[2Q + \frac{Q}{7R^3}(r^3-R^3)\right]\cdot\frac{1}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{1}{r^2}\left[2Q+\frac{Q}{7R^3}(r^3-R^3)\right]$$

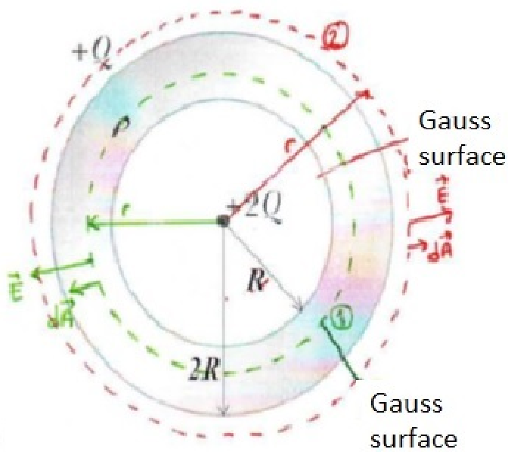
$$E = k\left(\frac{2Q}{r^2}+\frac{Qr}{7R^3}-\frac{Q}{7r^2}\right)$$

$$\boxed{E = \frac{kQ}{7}\left(\frac{13}{r^2}+\frac{r}{R^3}\right)}$$

for $r>2R$ zone (2)

$$E(4\pi r^2) = \frac{2Q+Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{3Q}{r^2} \quad , \quad \boxed{E = 3k\frac{Q}{r^2}}$$




$$q_{sphere} = \frac{\frac{4}{3}\pi(r^3-R^3).Q}{\frac{4}{3}\pi(7R^3)}$$

$$q_{sphere} = \frac{Q}{7R^3}(r^3-R^3)$$

volume of $\frac{4}{3}\pi\left[(2R)^3 - R^3\right]$ of spherical shell has Qcharga

" $\frac{4}{3}\pi\left[r^3-R^3\right]$ $q_{sphere}$

b) for $R < r < 2R$ zone (1)

inside conductor $E = 0$ ; $q_{in} = (q_{in})_{surface} + 2Q$

$q = -2Q + 2Q = 0$

$$E(4\pi r^2) = \frac{q_{in}}{\epsilon_0} = 0$$

$$\boxed{E = 0}$$

for $r > 2R$ zone (2)

$$E(4\pi r^2) = \frac{2Q + Q}{\epsilon_0}$$

$$\boxed{E = 3k\frac{Q}{r^2}}$$

7. A solid, insulating sphere of radius **R** has a nonuniform charge density $\rho = \alpha r$ and a total charge **+2Q** ($\alpha$ positive constant and $r$ radial distance from origin). Concentric with this sphere is a charged (**+4Q**), conducting shell sphere whose inner and outer radii are **2R** and **3R**, as shown in **Figure 5**.

**a)** Find α constant in terms of Q and R.

Find the magnitude of the electric field in the regions in terms of k, Q, r and R.

**b)** r<R

**c)** R<r<2R

**d)** 2R<r< 3R

**e)** r>3R

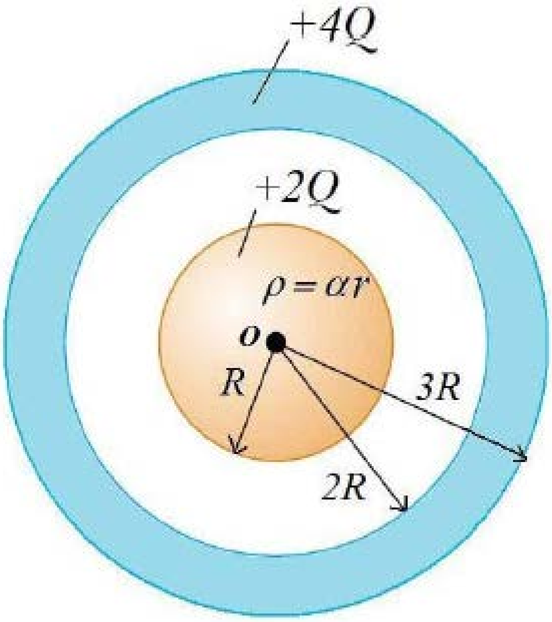
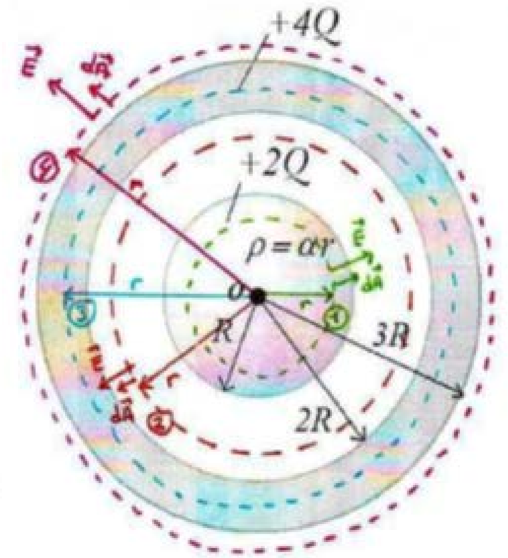



**Figure 6**




a) $dQ = \rho\, dV$  $V = \frac{4}{3}\pi r^3$

$\int_0^{2Q} dQ = \int_0^R (\alpha r) 4\pi r^2 dr$  $dV = 4\pi r^2 dr$

$Q\Big]_0^{2Q} = 4\pi\alpha\left[\frac{r^4}{4}\right]_0^R$

$2Q = \pi\alpha R^4$

$\boxed{\alpha = \frac{2Q}{\pi R^4}}$

b) $\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_i}{\epsilon_0}$  $q_i = \int \rho\, dV$

$E(4\pi r^2) = \frac{1}{\epsilon_0}\int_0^r (\alpha r) 4\pi r^2 dr$

$E(4\pi r^2) = \frac{1}{\epsilon_0} 4\pi\alpha \left[\frac{r^4}{4}\right]_0^r$

$E = \frac{1}{4\pi\epsilon_0}\,\frac{4\pi}{r^2}\,\frac{2Q}{\pi R^4}\cdot\frac{r^4}{4}$ , $\boxed{E = 2k\frac{Qr^2}{R^4}}$ $r<R$

c) $E(4\pi r^2) = \frac{2Q}{\epsilon_0}$

$E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$ , $\boxed{E = 2k\frac{Q}{r^2}}$ $R < r < 2R$

d) $E(4\pi r^2) = \frac{2Q - 2Q}{\epsilon_0}$ $q_i = 2Q + \underbrace{(q_i)_{\text{surface}}}_{-2Q}$

$\boxed{E = 0}$ $2R < r < 3R$

e) $E(4\pi r^2) = \frac{4Q + 2Q}{\epsilon_0}$

$\boxed{E = 6k\frac{Q}{r^2}}$ $r > 3R$

**8.** A point charge q locates at the centre of a cylinder with radius a and height 2h (see Figure 7). Show that the electric flux through the lateral surface of the cylinder is given by $\dfrac{\sqrt{2}}{2}\dfrac{q}{\varepsilon_0}$.

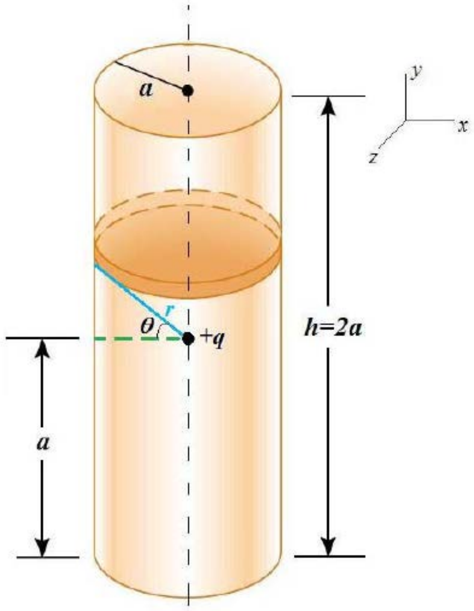
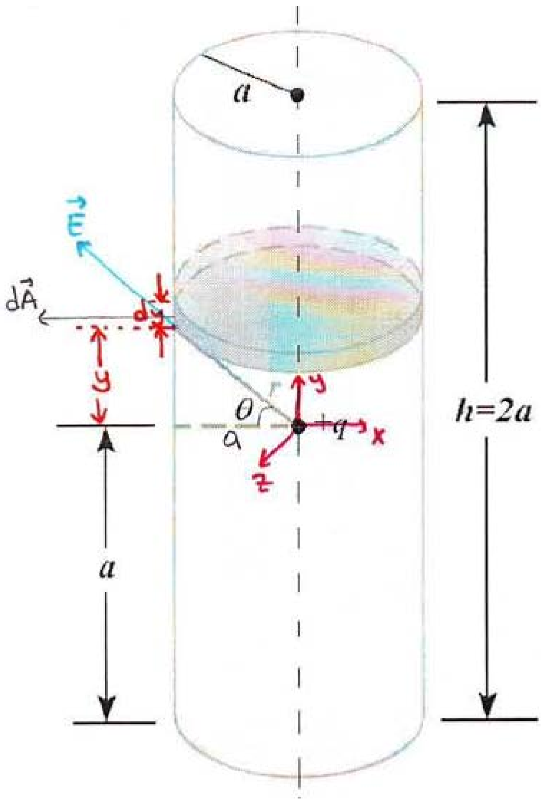



**Figure 7**




$$\Phi_E = \oint \vec{E}\cdot d\vec{A} \qquad \vec{E} = k\frac{q}{r^2}\hat{r}$$

$$\Phi_E = \oint E\, dA \cos\theta$$

$$dA = 2\pi a\, dy$$

$$\cos\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\theta}$$

$$tg\theta = \frac{y}{a} \Rightarrow y = a\, tg\theta$$

$$dy = a\sec^2\theta\, d\theta$$

$$\Phi_E = \int k\frac{q}{r^2}\, dA\cos\theta = \int k\frac{q}{r^2}\, 2\pi a\, dy\, \frac{a}{r}$$

$$\Phi_E = 2\pi a^2 kq \int_{-a}^{a} \frac{dy}{r^3} = 2\pi a^2 kq \int_{-a}^{a} \frac{dy}{\left(\frac{a}{\cos\theta}\right)^3}$$

$$\Phi_E = 2\pi a^2 kq \int \frac{\cos^3\theta \; a \sec^2\theta \, d\theta}{a^{3}}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\Phi_E = 2\pi kq \int_{-\pi/4}^{\pi/4} \cos\theta \, d\theta$$

$$\Phi_E = 2\pi kq \sin\theta \Big]_{-\pi/4}^{\pi/4}$$

**integral zone**

$y=-a$ ; $y = a\,tg\theta$

$-a = a\,tg\theta$

$tg\theta = -1$

$\theta = -\pi/4$

$y = a$ ; $y = a\,tg\theta$

$a = a\,tg\theta$

$tg\theta = 1$

$\theta = \pi/4$

$$\Phi_E = 2\pi kq \left[\sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)\right]$$

$$\Phi_E = 2\pi kq \left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right]$$

$$\Phi_E = 2\pi kq\sqrt{2}$$

$$\Phi_E = 2\pi \frac{1}{4\pi\epsilon_0} q\sqrt{2}$$

$$\boxed{\Phi_E = \frac{\sqrt{2}}{2}\frac{q}{\epsilon_0}}$$