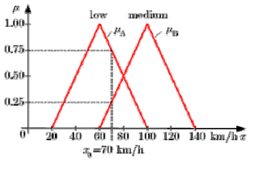
FUZZY-TO-CRISP CONVERSIONS

Lecture 05

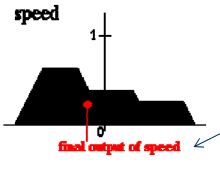
FUZZY-TO-CRISP CONVERSIONS

Fuzzification: Making a crisp quantity fuzzy.



Assignment of membership functions is the process of fuzzification

Defuzzification: Making a fuzzy quantity crisp.



Calculation of a crisp value from a fuzzy value

FROM FUZZY SETS TO CRISP SETS

LAMBDA-CUTS for fuzzy sets

$\underset{\sim}{A}$: a fuzzy set A

A_λ: Lambda-Cut set of A

A_λ: $\{x \mid \mu_{\underline{A}}(x) \geq \lambda\}$ where $0 \leq \lambda \leq 1$

The set A_λ is a crisp set.

FROM FUZZY SETS TO CRISP SETS

Example: X={a,b,c,d,e,f} and $\underset{\sim}{A}=\{\frac{1}{a}+\frac{0.9}{b}+\frac{0.6}{c}+\frac{0.3}{d}+\frac{0.01}{e}+\frac{0}{f}\}$

$\lambda = 1 \quad \rightarrow A_1=\{a\}$;

$\lambda = 0.8 \rightarrow A_{0.8}=\{a,b\}$;

$\lambda = 0.6 \rightarrow A_{0.6}=\{a,b,c\}$;

$\lambda = 0^+ \quad \rightarrow A_{0^+}=\{a,b,c,d,e\}$;

$\lambda = 0 \quad \rightarrow A_0=\{a,b,c,d,e,f\} = X$

Fuzzy notation of λ-Cut Sets:

$\underset{\sim}{A}_{0.6}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{0}{d}+\frac{0}{e}+\frac{0}{f}\}$

$\underset{\sim}{A}_{0.25}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{0}{e}+\frac{0}{f}\}$

FROM FUZZY SETS TO CRISP SETS

LAMBDA-CUT SET PROPERTIES

1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = A_\lambda \cup B_\lambda$
2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = A_\lambda \cap B_\lambda$
3. $(\bar{\underset{\sim}{A}})_\lambda \neq \bar{A}_\lambda$ except for λ=0.5
4. For any $\lambda \leq \alpha$ where $0 \leq \alpha \leq 1$, it is true that $A_\alpha \subseteq A_\lambda$ where A_0=X

FUZZY-TO-CRISP RELATIONS

LAMBDA-CUTS FOR FUZZY RELATIONS

$\underset{\sim}{R}$: A fuzzy relation

R_λ: λ-cut relation of $\underset{\sim}{R}$.

$R_\lambda = \{(x,y) \mid \mu_{\underset{\sim}{R}}(x,y) \geq \lambda\}$ for $0 \leq \lambda \leq 1$

FUZZY-TO-CRISP RELATIONS

Example: R = $\begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0.4 \\ 0 & 0.4 & 1 \end{bmatrix}$

$\lambda = 1 \rightarrow \mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\lambda = 0.25 \rightarrow \mathbf{R}_{0.25} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$;

$\lambda = 0.5 \rightarrow \mathbf{R}_{0.5} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\lambda = 0 \rightarrow \mathbf{R}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

FUZZY-TO-CRISP RELATIONS

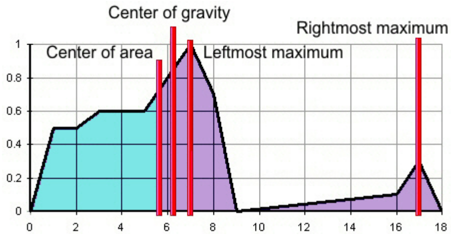
PROPERTIES:

1. $(\underset{\sim}{R} \cup \underset{\sim}{S})_\lambda = R_\lambda \cup S_\lambda$
2. $(\underset{\sim}{R} \cap \underset{\sim}{S})_\lambda = R_\lambda \cap S_\lambda$
3. $(\bar{\underset{\sim}{R}})_\lambda \neq \bar{R}_\lambda$
4. For any $\lambda \leq \alpha$ where $0 \leq \alpha \leq 1$, then $R_\alpha \subseteq R_\lambda$.

DEFUZZIFICATION TO SCALARS

DEFUZZIFICATION METHODS

There many defuzzification methods in the literature.

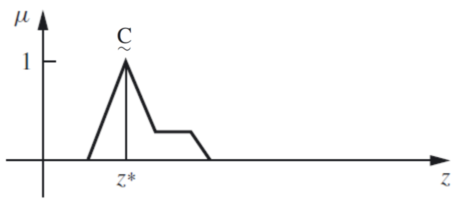


DEFUZZIFICATION TO SCALARS

1. MAX-MEMBERSHIP PRINCIPLE:

Also known as the height method, this scheme is limited to peaked output functions. This method is given by the algebraic expression where z^* is the defuzzified value.

$$\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z), \qquad \text{for all } z \in Z,$$



DEFUZZIFICATION TO SCALARS

2. FIRST (or Last) OF MAXIMA: (more than 1 maximum case)
This method uses the overall output or union of all individual output fuzzy sets C_k to determine the smallest value of the domain with maximized membership degree in C_k.

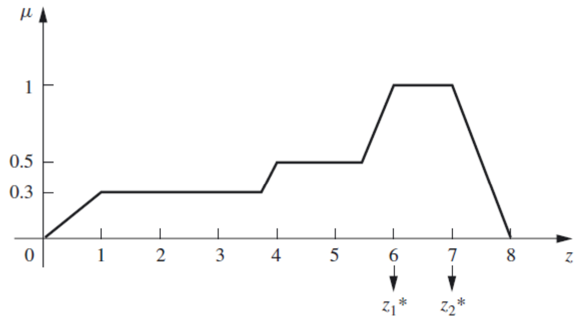
Height in the union: $$\mathrm{hgt}(\underset{\sim}{C}_k) = \sup_{z \in Z} \mu_{\underset{\sim}{C}_k}(z)$$

a) First of Maxima: $$z* = \inf_{z \in Z} \left\{ z \in Z \mid \mu_{\underset{\sim}{C}_k}(z) = \mathrm{hgt}(\underset{\sim}{C}_k) \right\}$$

b) Last of Maxima: $$z^* = \sup_{z \in Z} \left\{ z \in Z \mid \mu_{\underset{\sim}{C}_k}(z) = \mathrm{hgt}(\underset{\sim}{C}_k) \right\}$$

DEFUZZIFICATION TO SCALARS

Example:

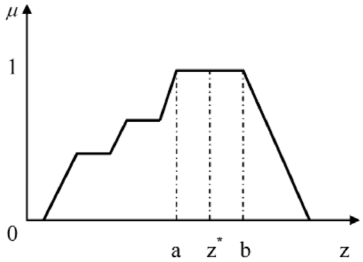


First of maxima solution ($z_1^* = 6$) and last of maxima solution ($z_2^* = 7$).

DEFUZZIFICATION TO SCALARS

3. MEAN-MAX MEMBERSHIP (Middle of Maxima) METHOD:
Similar to the first method. If there are more than one max point, take average of them.

Example:



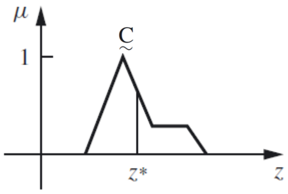
$$z^* = \frac{a+b}{2}$$

DEFUZZIFICATION TO SCALARS

4. CENTROID METHOD (Center of Area or Center of Gravity):

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz}$$

Gives the z point which is located at the center of gravity



This is the most widely used defuzzification method.

Centroid defuzzification method.

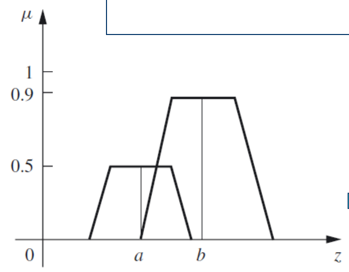
DEFUZZIFICATION TO SCALARS

5. WEIGHTED AVERAGE METHOD:

This method is only valid for symmetrical output membership functions. It is computationally very efficient.

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})}$$

Example:



$$z^* = \frac{a(0.5) + b(0.9)}{0{,}5 + 0{,}9}$$

DEFUZZIFICATION TO SCALARS

6. CENTER OF SUMS METHOD:

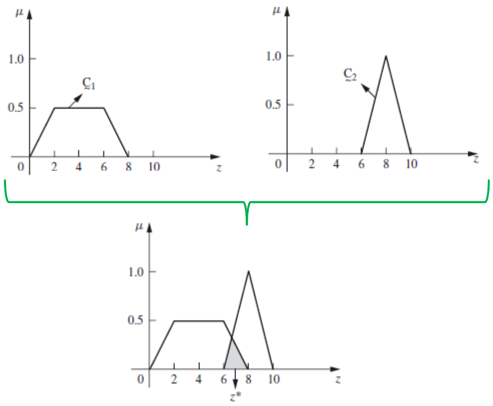
This process involves the algebraic sum of individual output fuzzy sets instead of their union. Two drawbacks to this method are that the intersecting areas are added twice, and the method also involves finding the centroids of the individual membership functions.

$$z^* = \frac{\sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z) \int_z \bar{z}\, dz}{\sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z) \int_z dz}$$

DEFUZZIFICATION TO SCALARS

Center of sums method:

Example:



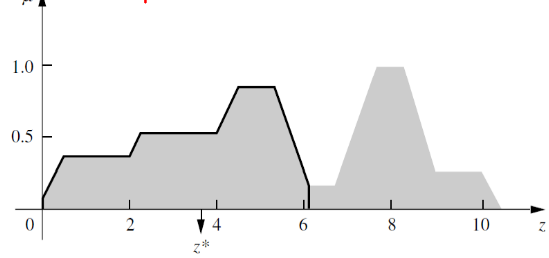
DEFUZZIFICATION TO SCALARS

7. CENTER OF LARGEST AREA METHOD:
If the output set has at least two convex sub-regions, then the center of gravity of the convex fuzzy subregion of the largest area is used.

$$z^* = \frac{\int \mu_{\underset{\sim}{C}_m}(z)\, z\, dz}{\int \mu_{\underset{\sim}{C}_m}(z)\, dz}$$

where C_m is the convex subregion that has the largest area making up C_k

Example:



FUZZY-TO-CRISP CONVERSIONS

- There are many other defuzzification methods available:
  - AI (adaptive integration)
  - BADD (basic defuzzification distributions)
  - BOA (bisector of area)
  - CDD (constraint decision defuzzification)
  - ECOA (extended center of area)
  - EQM (extended quality method)
  - FCD (fuzzy clustering defuzzification)
  - FM (fuzzy mean)
  - GLSD (generalized level set defuzzification)
  - ICOG (indexed center of gravity)
  - IV (influence value)
  - QM (quality method)
  - RCOM (random choice of maximum)
  - SLIDE (semi-linear defuzzification)

Homework

Homework:
See extra examples related to this subject from the textbook.