Fuzzy Logic

Introduction

Fuzzy Logic

- **Lecturer :** H. Serhan YAVUZ , hsyavuz@ogu.edu.tr
- **Text Book :** Fuzzy Logic with Engineering Applications, Timothy J. Ross (Also available online in OGU library)
- **Additional book :** Neuro-Fuzzy and Soft Computing, J. R. Jang, C. Sun.

Any other introduction to fuzzy logic book is also all right.

- **Turkish book:** Zekai Şen : " Bulanık Mantık ve Modelleme İlkeleri"

Fuzzy Logic

GRADING

30% ⇨ Midterm Exam
25% ⇨ Lab Performance
10% ⇨ Project
35% ⇨ Final Exam

- ❖ Closed Book Exams, no formula sheet, ≲90 mins
- ❖ Absenteeism : General rules will be obeyed
- ❖ Any question?

Fuzzy Logic

TERM SUBJECTS

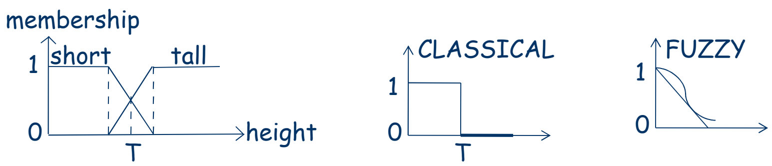
1. Introduction
2. Classical Sets and Fuzzy Sets
3. Classical Relations
4. Fuzzy Relations
5. Membership Functions
6. Fuzzy to Crisp Conversions
7. Fuzzy Arithmetic and Numbers
8. Fuzzy Extension Principle
9. Classical Logic and Fuzzy Logic
10. Fuzzy Rule Based Systems
11. Fuzzy Decision Making, Clustering
12. Fuzzy Pattern Recognition

Introduction

- ✓ 1965, Dr. Lotfi A. Zadeh
- ✓ The word fuzzy?
- ✓ What is Logic?
- ✓ Fuzzy Logic?
  - Fuzzy Logic provides a method to formalize reasoning when dealing with vague terms.
  - Not every decision is either true or false. Fuzzy Logic allows for membership functions, or degrees of truthfulness and falsehood.
- ✓ Bart Kosko example:
  - How many of you are Male (or female)? This distinction is an easy choice to make a decision.
  - How many of you are you are comfartable with his/her job or life?. This is not so easy to decide as yes or no.

Introduction

- ✓ Classical Logic : TRUE or FALSE
- ✓ Fuzzy Logic : How much TRUE and/or how much FALSE
- ✓ Example :
  - 50% full glass
  - Ali 1.69m, Veli 1.71m. T=1.70 m → Ali is short, Veli is tall.
- ✓ Fuzzy Membership : Degree of uncertainty



Introduction

✓ Fuzzy IF-THEN Rules
IF "Service" is "good" THEN "tip" is "good"
IF "Service" is "good" AND "food" is "normal" THEN "tip" is "average"

✓ Fuzzy Logic Based Machines/Applications
- Sendai Subway, Japon
- Cement Factory, Denmark
- Photographing machines, air conditiones, ABS/Cruise Control, elevators, washing machines, cookers, video game artificial intelligence, pattern recognition, etc.

Classical Sets & Fuzzy Sets

Lecture 02

Classical & Fuzzy Sets

Universe of discourse: universe of all available information on a given problem.

Classical set: A set having crisp boundaries (either it includes an element or not).

Fuzzy set: A set whose elements have membership degrees.

Classical set Examples

Classical sets are also called *crisp* (sets).

Lists: A = {apples, oranges, cherries, mangoes}

$A = \{a_1, a_2, a_3\}$

A = {2, 4, 6, 8, …}

Formulas: A = {x | x is an even natural number}

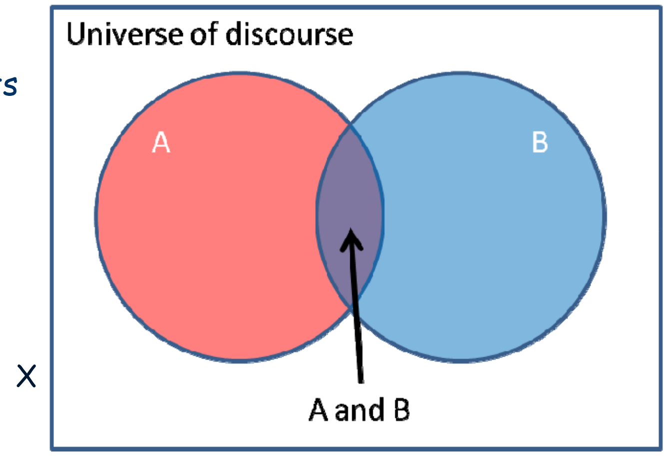
A = {x | x = 2n, n is a natural number}

Membership or characteristic function

$$X_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Classical Sets

Example:
two classical sets A and B with a universe of discourse X



Classical Sets

Classical Set Theory

$x \in X \Rightarrow$ x belongs to X
$x \in A \Rightarrow$ x belongs to A
$x \notin A \Rightarrow$ x does not belong to A

For sets A and B on X
(X: universe of discourse)

Classical Sets

$A \subset B \Rightarrow$ A is fully in B (if $x \in A$, then $x \in B$)

$A \subseteq B \Rightarrow$ A is contained in or is equivalent to B

$A = B \Rightarrow A \subseteq B$ and $B \subseteq A$

$\emptyset$: null set (a set containing no elements)

Classical Sets

Operations on Classical Sets

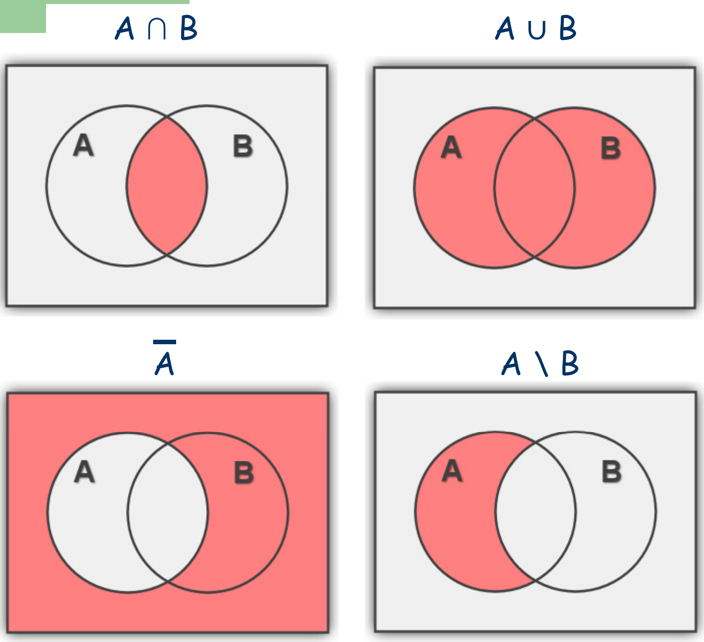
Let A, B be two sets on the universe X;

Union: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

Intersection: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

Complement: $\bar{A} = \{ x \mid x \notin A, x \in X \}$

Difference: $A \setminus B = \{ x \mid x \in A \text{ and } x \notin B \text{ with } x \in X \}$



Classical Sets

Properties of Classical Sets

Commutativity:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Associativity:

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Classical Sets

Properties of Classical Sets – ctd.

Distributivity:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotency:

$A \cup A = A$ $\quad$ $A \cap A = A$

Classical Sets

Properties of Classical Sets – ctd.

Identity:

$A \cup \emptyset = A$ $A \cap \emptyset = \emptyset$

$A \cap X = A$ $A \cup X = X$

Transitivity:

If $A \subseteq B \subseteq C$, then $A \subseteq C$

Classical Sets

Properties of Classical Sets – ctd.

Involution:

$\overline{\overline{A}} = A$

Law of the excluded middle:

$A \cup \overline{A} = X$

Classical Sets

Properties of Classical Sets

Law of the contradiction:

$$A \cap \overline{A} = \emptyset$$

De Morgan's Law:

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

Classical & Fuzzy Sets

Fuzzy Sets

A fuzzy set is a set containing elements that have varying degrees of membership in the set. Elements in a fuzzy set can also be members of other fuzzy sets on the same universe.

Notation $\rightarrow$ $\underset{\sim}{A}$: fuzzy set A

Classical & Fuzzy Sets

Fuzzy Sets

Each element in is defined by its membership value.

$$\mu_{\underset{\sim}{A}}(x) \in [\,0, 1\,]$$

A fuzzy set may be defined in either continuous or discrete universe.

Classical & Fuzzy Sets

Fuzzy Sets

X : Universe of discourse X ={ x }

$\underset{\sim}{A}$: Fuzzy set A.

$\mu_{\underset{\sim}{A}}(x)$: Membership value of x to $\underset{\sim}{A}$

$\underset{\sim}{A} = \{ (\, x, \mu_{\underset{\sim}{A}}(x)\,) \mid x \in X \}$

$\mu_{\underset{\sim}{A}}(x) \in [\,0, 1\,]$

Fuzzy Sets

Fuzzy Sets

A discrete fuzzy set notation:

$$\underset{\sim}{A} = \{\frac{\mu(x_1)}{x_1} + \frac{\mu(x_2)}{x_2} + ... + \frac{\mu(x_n)}{x_n}\}$$

(not a division)

(not a conventional sum)

A continuous fuzzy set:

$$\underset{\sim}{A} = \{ \int \frac{\mu(x)}{x} \}$$

(not an integral)

Fuzzy Sets

Example : A 6-people family. Ages of the family members are:

Ahmet : 52	Fatma :45	Mithat :27
Dilara : 25	Nuray : 19	Murat : 3

Develop a dicsrete fuzzy set “Old” by your own intuition.

Fuzzy Sets

Solution

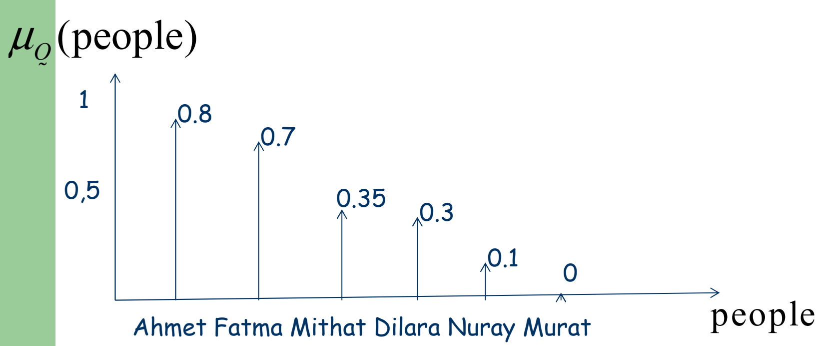
X : Universe os discourse (People)

X = { Ahmet, Fatma, Mithat, Dilara, Nuray, Murat }

$\underset{\sim}{Q}$: Fuzzy set "Old"

$$\underset{\sim}{Q} = \left\{ \frac{0.8}{\text{Ahmet}}, \frac{0.7}{\text{Fatma}}, \frac{0.35}{\text{Mithat}}, \frac{0.3}{\text{Dilara}}, \frac{0.1}{\text{Nuray}} \right\}$$

Fuzzy Sets



Membership Plot

Fuzzy Sets

Example: Construct a continuous fuzzy set "young".

Solution

Step-1 :Universe of discourse

X = [0, 120] (Ages of people)

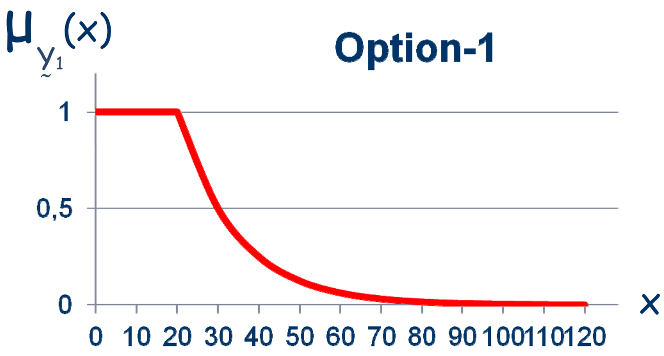
Fuzzy Sets

Step-2 : Fuzzy set "young"

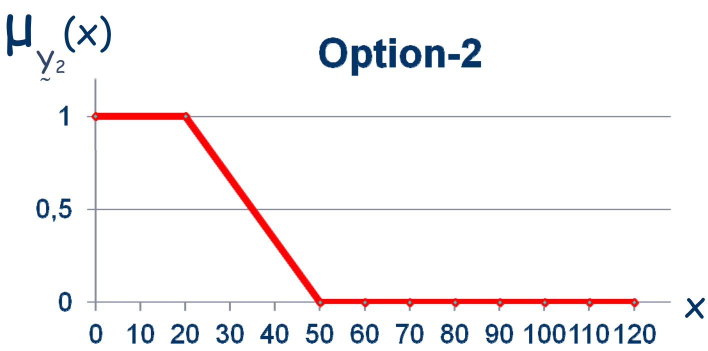
$\underset{\sim}{Y} \rightarrow$ What do we need?

Assign membership values.

Fuzzy Sets



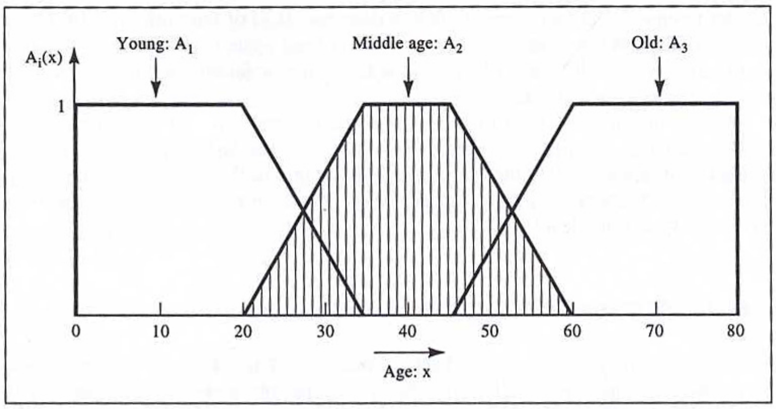
Fuzzy Sets



Fuzzy Sets

$$\mu_{\underset{\sim}{y_2}}(x) = \begin{cases} 1 & , \quad 0 \leq x \leq 20 \\ -x/30 + 5/3 & , \quad 20 \leq x \leq 50 \\ 0 & , \quad 50 \leq x \leq 120 \end{cases}$$

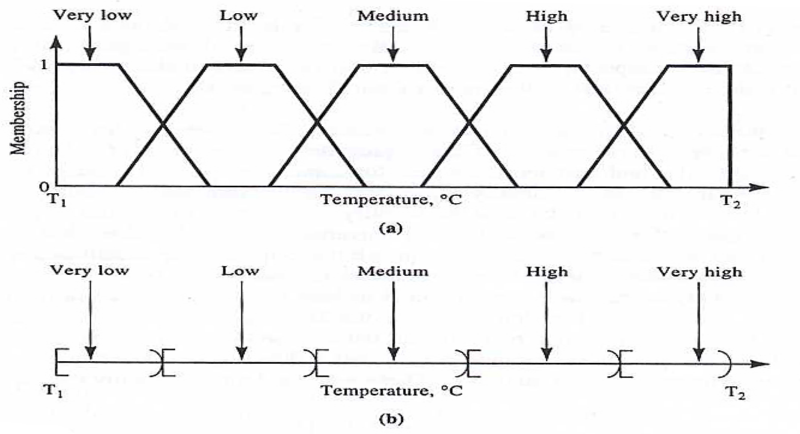
Example: 3 fuzzy sets: young, middle aged, old



Membership functions representing the concepts of a young, middle-aged, and old person.

(figure from the book: Klir&Yuan)

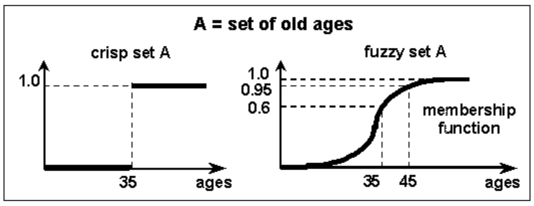
Example: Fuzzy sets *vs.* classical sets about the temperature

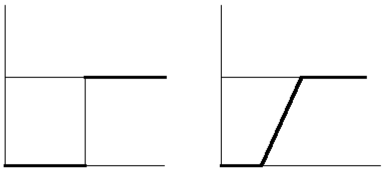


Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

(figure from the book: Klir&Yuan)

Example: Fuzzy sets *vs.* classical sets





A crisp set is a binary representation, but a fuzzy set is a kind of more smooth representation.

Fuzzy Sets

Operations on Fuzzy Sets

Let $\underset{\sim}{A}$, $\underset{\sim}{B}$ be two fuzzy sets;

Complement: $\mu_{\bar{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$

Fuzzy Sets

Operations on Fuzzy Sets

Union: $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$ ⇐ S-norm (T-conorm)

Intersection:

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x) \Leftarrow \text{T-norm}$$

Fuzzy Sets

Definition (T-norm):

T-norm operator is used for intersection

$T : [0,1] \times [0,1] \rightarrow [0,1]$

T-norm operator is denoted by T(.,.)

Fuzzy Sets

Any operator which satisfies the following four conditions can be called as a **T-norm** operator;

(1) $T(0,0) = 0$; $T(a,1) = T(1,a) = a$

(2) $a \leq c$ and $b \leq d \Rightarrow T(a,b) \leq T(c,d)$

(3) $T(a,b) = T(b,a)$

(4) $T(a, T(b,c)) = T(T(a,b),c)$

Fuzzy Sets

Some Famous T-norm Operators

Minimum : $T_{min}(a,b) = \min(a,b)$

Arithmetic Product : $T_{ap}(a,b) = ab$

Bounded Product : $T_{bp}(a,b) = 0 \vee (a+b-1)$

Fuzzy Sets

Drastic Product :

$$T_{dp}(a,b) = \begin{cases} a, & b = 1 \\ b, & a = 1 \\ 0, & a,b < 1 \end{cases}$$

Fuzzy Sets

We will use "**minimum**" operator as the T-norm operator.

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x) = \min(\mu_{\underset{\sim}{A}}, \mu_{\underset{\sim}{B}})$$

Fuzzy Sets

Definition (S-norm):

S-norm operator is used for union,

$S : [0,1] \times [0,1] \rightarrow [0,1]$

S-norm (or T-conorm) operator is denoted by S(.,.)

Fuzzy Sets

Any operator which satisfies the following four conditions can be called as an **S-norm** operator:

(1) $S(1,1) = 1$; $S(a,0) = S(0,a) = a$

(2) $a \leq c$ and $b \leq d \Rightarrow S(a,b) \leq S(c,d)$

(3) $S(a,b) = S(b,a)$

(4) $S(a, S(b,c)) = S(S(a,b),c)$

Fuzzy Sets

Some Famous S-norm Operators

Maximum : $S_{max}(a,b) = \max(a,b)$

Arithmetic Product : $S_{ap}(a,b) = a + b - ab$

Bounded Product : $S_{bp}(a,b) = 1 \wedge (a+b)$

Fuzzy Sets

Drastic Product :

$$S_{dp}(a,b) = \begin{cases} a, & b = 0 \\ b, & a = 0 \\ 1, & a,b > 0 \end{cases}$$

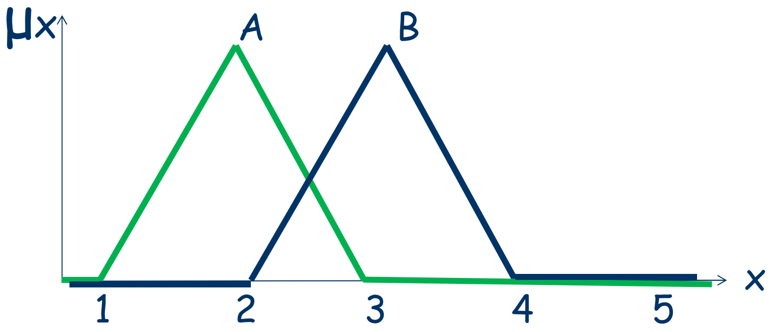
Fuzzy Sets

We will use "**maximum**",

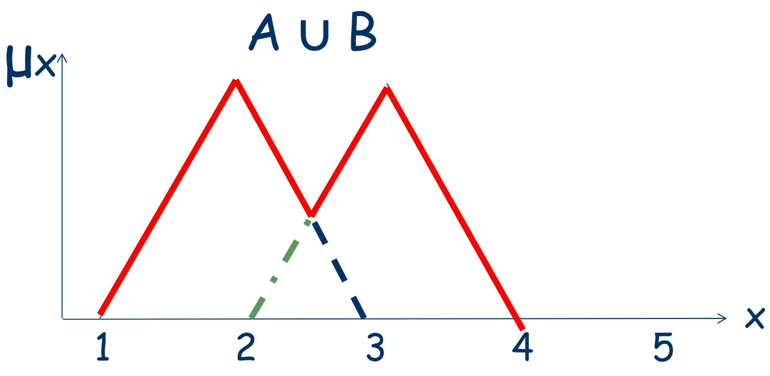
$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x) = \max(\mu_{\underset{\sim}{A}}, \mu_{\underset{\sim}{B}})$$

Fuzzy Sets

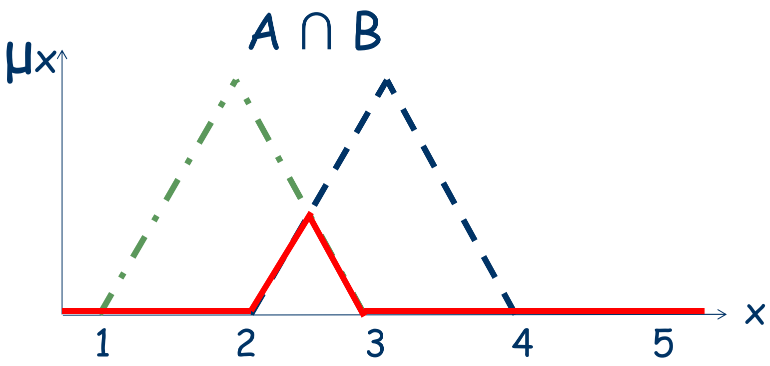
Example:



Fuzzy Sets



Fuzzy Sets



Fuzzy Sets

Properties of Fuzzy Sets

Associativity:

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C}$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}$$

Fuzzy Sets

Distributivity:

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C})$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{A} \cap \underset{\sim}{C})$$

Fuzzy Sets

Idempotency:

$$\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A} \qquad \underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$$

Identity:

$$\underset{\sim}{A} \cup \emptyset = \underset{\sim}{A} \qquad \underset{\sim}{A} \cap \emptyset = \emptyset$$

$$\underset{\sim}{A} \cap X = \underset{\sim}{A} \qquad \underset{\sim}{A} \cup X = X$$

Fuzzy Sets

Transitivity:

If $\underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}$, then $\underset{\sim}{A} \subseteq \underset{\sim}{C}$

Involution:

$\overline{\overline{\underset{\sim}{A}}} = \underset{\sim}{A}$

Fuzzy Sets

- End of the lecture. Any questions?
- Please check DYS each week for any uploaded supplementaries .

http://enf.ogu.edu.tr/golddys/