Haberleşme Sistemlerinde

Temel Bilgiler

Güz 2011-12

Tuncay ERTAŞ

1. Hafta

Temel Bilgiler

Bölüm I
Sinyaller ve Sistemler

- **Sinyaller ve Sınıflandırılması**
- **Güç ve Enerji**
- **Fourier Serileri**
- **Fourier Transformu ve Özellikleri**
- **Dirac Delta Fonksiyonu**

Temel Bilgiler

Sinyaller

- Bir **g(*t*)** ***sinyali*** zamanın bir fonksiyonudur
  - Gerilim v(*t*) veya akım i(*t*) olabilir
- Bir sinyalin ***fiziksel olarak gerçeklenebilmesi için,*** *sinyal* **:**
  - Zamanda sınırlı olmalıdır.
  - Bant genişliği sonlu olmalıdır.
  - Zamanda sürekli olmalıdır.
  - Aldığı değerler sonlu olmalıdır.
  - Gerçel değerli olmalıdır.

Temel Bilgiler

Periyodik ve Aperiyodik Sinyaller

$$g(t) = g(t + T_o) \quad , \quad \forall t$$

Şeklinde ifade edilebilen sinyallere ***periyodik sinyal*** denir.

- Aksi takdirde g(*t*) ***aperiyodiktir***.
- Yukarıdaki bağıntıyı sağlayan en küçük T_0 değerine ***sinyalin temel periyodu*** denir.

Temel Bilgiler

Güç: Anlık ve Normalize

- Bir devrede ***Anlık Güç*** : $p(t) = v(t)i(t)$
- *Ohm kanunundan,* $p(t) = \frac{v^2(t)}{R} = i^2(t)R$
- ***Anlık normalize güç*** R=1 Ohm alınarak bulunur:

$$p(t) = v^2(t) = i^2(t)$$

- g(t) bir gerilim veya bir akım olabileceğinden ***g(t) sinyalinin anlık normalize gücü***:

$$p(t) = g^2(t)$$

olarak yazılır.

Temel Bilgiler

Ortalama Normalize Güç

- Bir sinyalin ***ortalama normalize gücü*** anlık normalize gücünün zaman ortalaması alınarak bulunur:

$$P = \left\langle g^2(t) \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt$$

  - burada $\langle \cdot \rangle$ ***zaman ortalaması operatörüdür***.

Temel Bilgiler

Güç Sinyalleri

- Ortalama normalize gücü sıfırdan farklı ve sonlu olan sinyale ***güç sinyali*** denir

$$0 < P < \infty$$

- Güç sinyalleri fiziksel olarak gerçeklenemez!
  - Çünkü bu sinyaller ya sonsuza kadar devam eder ya da bir anda sonsuz bir değer alırlar. Dolayısı ile enerjileri sonsuzdur!
- Güç sinyalleri periyodik veya aperiyodik olabilirler.

Temel Bilgiler

Enerji Sinyalleri

- Bir sinyalin ***normalize enerjisi***

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} g^2(t)\,dt$$

olarak tanımlanır.

- Enerjisi sıfırdan farklı ve sonlu olan sinyallere ***enerji sinyali*** denir. Öyle ki,

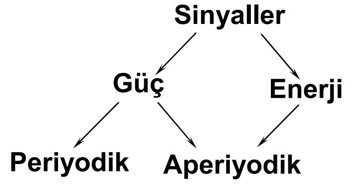
$$0 < E < \infty$$

- Dikkat: Enerji sinyallerinin ortalama gücü sıfırdır!

Temel Bilgiler

Sinyallerin Sınıflandırılması

- Bir sinyal ***güç*** veya ***enerji*** sinyali olarak sınıflandırılır
  - Güç Sinyali: $0 < P < \infty$
  - Enerji Sinyali: $0 < E < \infty$
- Güç sinyalleri ***periyodik*** veya ***aperiyodik*** olabilir
- Enerji sinyalleri daima ***aperiyodiktir***.



Temel Bilgiler

Periyodik Sinyallerin Gücü

- Periyodik bir sinyalin ortalama normalize gücü, bir periyot boyunca anlık normalize gücünün ortalamasıdır.

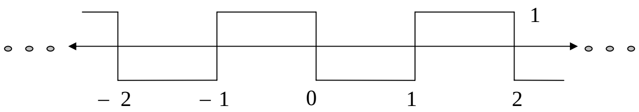
$$P = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} g^2(t)dt$$

- Dikkat: Limit operatörüne gerek olmadığını fark ediniz!

Temel Bilgiler

Örnek

- Aşağıdaki g(*t*) sinyalinin ortalama normalize gücünü bulunuz.



$$P = \frac{1}{2}\int_{-1}^{1} 1\,dt = 1\,Watt$$

- $A\cos(2\pi f_0 t)$ sinyalinin ortalama normalize gücünü bulunuz.

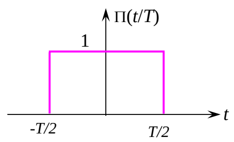
$$P = \frac{1}{T}\int_0^T A^2\cos^2(2\pi f_0 t)\,dt \qquad T = 1/f_0$$

$$= \frac{A^2}{T}\int_0^T \frac{1+\cos(4\pi f_0 t)}{2}\,dt = \frac{A^2}{2}\,Watt$$

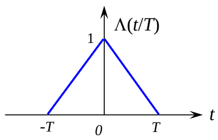
Temel Bilgiler

Bazı Önemli Sinyaller

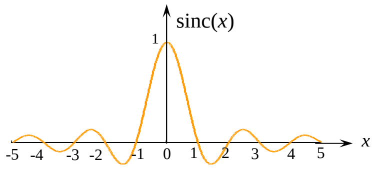
$$\Pi\left(\frac{t}{T}\right) = rect\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \le \frac{T}{2} \\ 0 & \text{if } |t| > \frac{T}{2} \end{cases}$$



$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T} & \text{if } |t| \le T \\ 0 & \text{if } |t| > T \end{cases}$$



$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



Temel Bilgiler

Fourier Serisi

Periyodu T_0 olan bir $g_p(t)$ sinyali için,

$$g_p(t) = a_o + 2\sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi nt}{T_o}\right) + b_n \sin\left(\frac{2\pi nt}{T_o}\right)\right]$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\,dt$$

$$a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\cos\left(\frac{2\pi nt}{T_0}\right)dt, \quad n = 1, 2, 3, \cdots$$

$$b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\sin\left(\frac{2\pi nt}{T_0}\right)dt, \quad n = 1, 2, 3, \cdots$$

Temel Bilgiler

Kompleks Fourier Serisi

$$g_p(t) = a_o + \sum_{n=1}^{\infty}\left[(a_n - jb_n)\exp\left(\frac{j2\pi nt}{T_o}\right) + (a_n + jb_n)\exp\left(-\frac{j2\pi nt}{T_o}\right)\right]$$

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{j2\pi nt}{T_o}\right)$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\exp\left(-\frac{j2\pi nt}{T_0}\right)dt, \quad n = 0, \pm 1, \pm 2, \cdots$$

$$c_n = \begin{cases} a_n - jb_n, & n > 0 \\ a_o \quad , & n = 0 \\ a_n + jb_n, & n < 0 \end{cases}$$

Genel olarak, c_n katsayıları komplekstir. Dolayısı ile, $c_n = |c_n|\exp[j\arg(c_n)]$

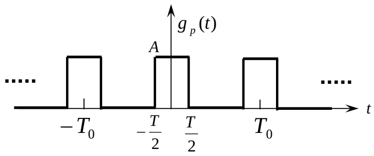
Gerçel Değerli Sinyaller için, $c_{-n} = c_n^*$ **, dolayısı ile de**

$|c_{-n}| = |c_n|$ **ve** $\arg(c_{-n}) = -\arg(c_n)$

Temel Bilgiler

Örnek

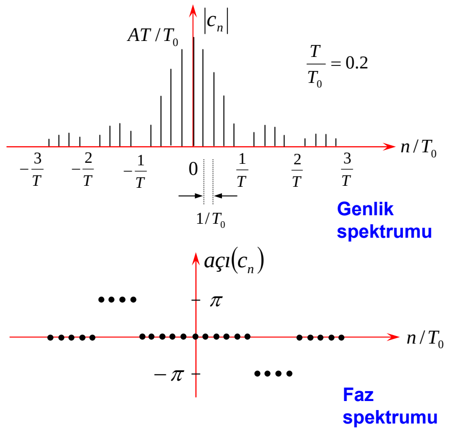
$$g_p(t)=\begin{cases}A, & -\frac{T}{2}\le t\le \frac{T}{2}\\ 0, & \text{periyodun kalanı için}\end{cases}$$



$$c_n=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\exp\left(-\frac{j2\pi nt}{T_0}\right)dt$$

$$=\frac{A}{n\pi}\sin\left(\frac{n\pi T}{T_o}\right),\quad n=0,\pm1,\pm2,....$$

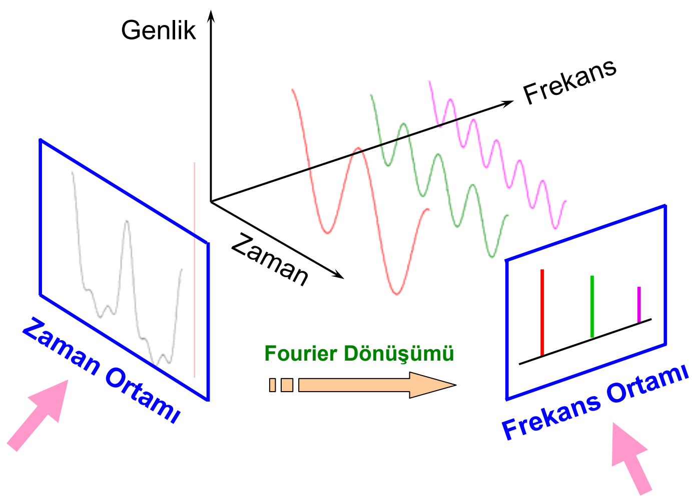
$$=\frac{TA}{T_o}\text{sinc}\left(\frac{nT}{T_o}\right)$$



DİKKAT!
Spektrum ayrık
Darbe parametrelerinin etkisi
Faz tek, genlik ise çift simetriye sahip

Temel Bilgiler

Sinyal ve Frekans Spektrumu



Temel Bilgiler

Fourier Transformu

- Bir ***aperiyodik*** g(t) sinyali için

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)dt$$ Fourier Transformu

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)df$$ Ters Fourier Transformu

$g(t) \rightleftarrows G(f)$ $F[g(t)] = G(f)$ $F^{-1}[G(f)] = g(t)$

g(t) nin Fourier transformuna g(t) nin *Spektrumu* da denir.

Fourier Transformu genellikle komplekstir: $G(f) = |G(f)|\exp[j\theta(f)]$

Gerçel değerli bir sinyal için: $G(f) = G^*(-f)$

Dolayısı ile,

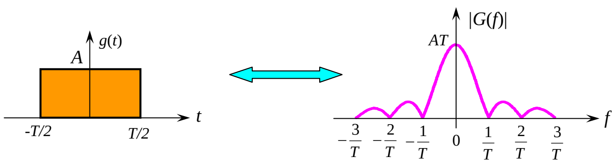
$|G(-f)| = |G(f)|$ çift simetri

$\theta(-f) = -\theta(f)$ tek simetri

Temel Bilgiler

Örnek

$$A\,\text{rect}\left(\frac{t}{T}\right) \Leftrightarrow AT\,\text{sinc}(fT)$$

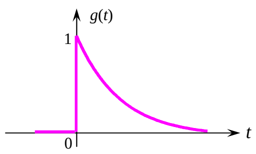


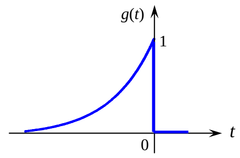
$g(t) = A\,rect\left(\frac{t}{T}\right) = A\prod\left(\frac{t}{T}\right)$

$$G(f) = \int_{-T/2}^{T/2} A\exp(-j2\pi ft)dt$$
$$= AT\left[\frac{\sin(\pi fT)}{\pi fT}\right]$$
$$= AT\text{sinc}(fT)$$

Örnek

Temel Bilgiler





$$g(t) = \exp(-t)u(t)$$

$$G(f) = \int_{-\infty}^{\infty} \mathbf{exp(-}t\mathbf{)exp}(-j2\pi ft)dt$$

$$= \int_{0}^{\infty} \mathbf{exp}[-(1+j2\pi f)t]dt$$

$$= \frac{1}{1+j2\pi f}$$

$$\exp(-t)u(t) \Leftrightarrow \frac{1}{1+j2\pi f}$$

$$g(t) = \exp(t)u(-t)$$

$$G(f) = \int_{-\infty}^{\infty} \mathbf{exp(}t\mathbf{)exp}(-j2\pi ft)dt$$

$$= \int_{-\infty}^{0} \mathbf{exp}[(1-j2\pi f)t]dt$$

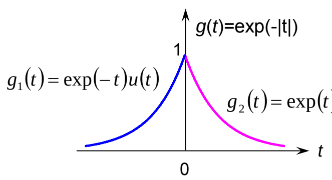
$$= \frac{1}{1-j2\pi f}$$

$$\exp(t)u(-t) \Leftrightarrow \frac{1}{1-j2\pi f}$$

Doğrusallık Özelliği

Temel Bilgiler

$$ag_1(t)+bg_2(t) \Leftrightarrow aG_1(f)+bG_2(f)$$



$$g(t) = \exp(-|t|) = g_1(t) + g_2(t)$$

$$G(f) = \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f}$$

$$g_1(t) \Leftrightarrow \frac{1}{1+j2\pi f}$$

$$g_2(t) \Leftrightarrow \frac{1}{1-j2\pi f}$$

$$\exp(-|t|) \Leftrightarrow \frac{2}{1+(2f\pi)^2}$$

Genleştirme Özelliği

Temel Bilgiler

$$g(at) \Leftrightarrow \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

$$F[g(at)] = \int_{-\infty}^{\infty} g(at)\mathbf{exp}(-j2\pi ft)dt$$

$\tau = at$ yazarak

$$F[g(at)] = \frac{1}{a}\int_{-\infty}^{\infty} g(\tau)\mathbf{exp}\left(-j2\pi\left(\frac{f}{a}\right)\tau\right)dt$$

$$= \frac{1}{a} G\left(\frac{f}{a}\right)$$

Örnek

$$g(t) = \begin{cases} \exp(-at), & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases} \qquad G(f) = \frac{1}{a(1 + j2\pi f / a)}$$

Dualite Özelliği

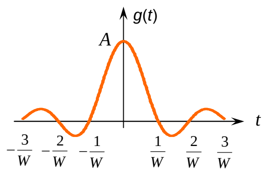
Temel Bilgiler

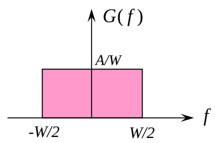
$$G(t) \Leftrightarrow g(-f)$$

$$g(-t) = \int_{-\infty}^{\infty} G(f)\mathbf{exp}(-j2\pi ft)df$$ **ve *t* ile *f* birbirinin yerine yazılırsa,**

$$g(-f) = \int_{-\infty}^{\infty} G(t)\exp(-j2\pi ft)dt$$

Örnek $g(t) = A\mathrm{sinc}(Wt) \Leftrightarrow \frac{A}{W} rect\left(\frac{f}{W}\right)$





Temel Bilgiler

Zamanda Öteleme Özelliği

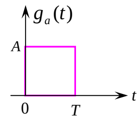
$$g(t-t_o) \Leftrightarrow G(f)\exp(-j2\pi f t_o)$$

$\tau = t - t_0$ **yazılırsa,**

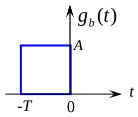
$$F[g(t-t_o)] = \mathbf{exp}(-j2\pi f t_o)\int_{-\infty}^{\infty} g(\tau)\,\mathbf{exp}(-j2\pi f \tau)\,d\tau = \mathbf{exp}(-j2\pi f t_o)\,G(f)$$

Örnek

$$G_a(f) = AT\mathrm{sinc}(fT)\exp(-j\pi f T)$$



$$G_b(f) = AT\mathrm{sinc}(fT)\exp(j\pi f T)$$



Temel Bilgiler

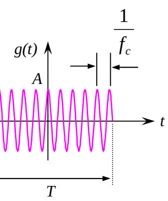
Frekansta Öteleme Özelliği

$$\exp(j2\pi f_c t)g(t) \Leftrightarrow G(f-f_c)$$

$$F[\exp(j2\pi f_c t)g(t)] = \int_{-\infty}^{\infty} g(t)\exp[-j2\pi t(f-f_c)]\,dt = G(f-f_c)$$

Örnek $g(t) = A\,rect\left(\frac{t}{T}\right)\cos(2\pi f_c t)$

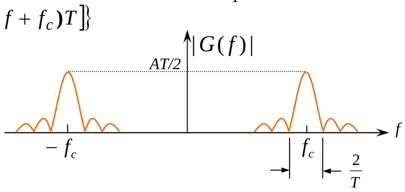
$f_c T >> 1$



$$\cos(2\pi f_c t) = \frac{1}{2}\left[\exp(j2\pi f_c t) + \exp(-j2\pi f_c t)\right]$$

$$G(f) = \frac{AT}{2}\{\mathbf{sinc}[(f-f_c)T] + \mathbf{sinc}[(f+f_c)T]\}$$

$$G(f) \approx \begin{cases} \frac{AT}{2}\mathrm{sinc}[(f-f_c)T], & f>0 \\ \frac{AT}{2}\mathrm{sinc}[(f+f_c)T], & f<0 \end{cases}$$



g(t) ve G(f) Altında Kalan Alan

Temel Bilgiler

$$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$$

$$g(0) = \int_{-\infty}^{\infty} G(f)\,df$$

Örnek $A\,\mathrm{sinc}(2Wt) \Leftrightarrow \frac{A}{2W}\,rect\left(\frac{f}{2W}\right)$

f=0 **yazılırsa,**

$$\int_{-\infty}^{\infty} A\,\mathrm{sinc}(2Wt)\,dt = \frac{A}{2W}$$

Ayrıca, özel olarak A=1 ve 2W=1 alınırsa, $\int_{-\infty}^{\infty} \mathrm{sinc}(t)\,dt = 1$

Zamanda Türev Özelliği

Temel Bilgiler

$$\frac{d}{dt}g(t) \Leftrightarrow j2\pi f G(f)$$

n. türev için ise,

$$\frac{d^n}{dt^n}g(t) \Leftrightarrow (j2\pi f)^n G(f)$$

Zamanda İntegral Özelliği

Temel Bilgiler

$$\int_{-\infty}^{t} g(\tau)\,d\tau \Leftrightarrow \frac{1}{j2\pi f}G(f)$$

G(0)=0 için

g(*t*) $g(t)=\frac{d}{dt}\left[\int_{-\infty}^{t} g(\tau)\,d\tau\right]$ **şeklinde ifade edilip, türev özelliği kullanılırsa,**

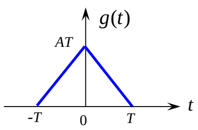
$$G(f) = j2\pi f\left\{F\left[\int_{-\infty}^{t} g(\tau)\,d\tau\right]\right\}$$

$G(0)\neq 0$ **için ise,**

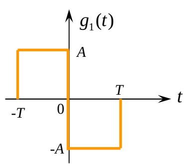
$$\int_{-\infty}^{t} g(\tau)\,d\tau \Leftrightarrow \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$$

Örnek

Temel Bilgiler



g(t) üçgen darbesinin Fourier transformunu bulunuz.



$$G_1(f) = AT\,\mathbf{sinc}(fT)\left[\mathbf{exp}(j\pi fT)-\mathbf{exp}(-j\pi fT)\right]$$
$$=2jAT\,\mathbf{sinc}(fT)\mathbf{sin}(\pi fT)$$

g(t), g_1(t) nin integrali olduğundan,

$$G(f)=\frac{1}{j2\pi f}G_1(f)$$
$$=AT\frac{\mathbf{sin}(\pi fT)}{\pi f}\mathbf{sinc}(fT)$$
$$=AT^2\,\mathbf{sinc}^2(fT)$$

Kompleks Eşlenik Özelliği

Temel Bilgiler

$$g^*(t) \Leftrightarrow G^*(-f)$$

$$g(t) = \int_{-\infty}^{\infty} G(f)\mathbf{exp}(j2\pi ft)df \qquad g^*(t) = \int_{-\infty}^{\infty} G^*(f)\mathbf{exp}(-j2\pi ft)df$$

$$= -\int_{\infty}^{-\infty} G^*(-f)\mathbf{exp}(j2\pi ft)df$$

$$= \int_{-\infty}^{\infty} G^*(-f)\mathbf{exp}(j2\pi ft)df$$

Örnek

Temel Bilgiler

$$g(t) = \mathrm{Re}[g(t)] + j\,\mathrm{Im}[g(t)] \qquad g^*(t) = \mathrm{Re}[g(t)] - j\,\mathrm{Im}[g(t)]$$

$$\mathrm{Re}[g(t)] = \frac{1}{2}[g(t) + g^*(t)] \qquad \mathrm{Im}[g(t)] = \frac{1}{2j}[g(t) - g^*(t)]$$

$$\boxed{\mathrm{Re}[g(t)] \Leftrightarrow \frac{1}{2}[G(f) + G^*(-f)]} \qquad \boxed{\mathrm{Im}[g(t)] \Leftrightarrow \frac{1}{2j}[G(f) - G^*(-f)]}$$

Çarpma ve Konvolüsyon Özelliği

Temel Bilgiler

$$g_1(t)g_2(t) \Leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$$

$$g_1(t)g_2(t) \Leftrightarrow G_1(f) \otimes G_2(f)$$

Zamanda Çarpma

$$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \Leftrightarrow G_1(f)G_2(f)$$

$$g_1(t) \otimes g_2(t) \Leftrightarrow G_1(f)G_2(f)$$

Zamanda Konvolüsyon

Dirac Delta Fonksiyonu

Temel Bilgiler

- ***Dirac delta fonksiyonunun*** tanımı:

$$\delta(t) = 0, \quad t \neq 0 \text{ için}, \quad ve \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

- ***İmpuls*** olarak da bilinir.
- **Özellikleri:**

$$\delta(-t) = \delta(t)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$g(t)\delta(t-t_0) = g(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} g(t)\delta(t)dt = g(0)$$

$$\int_{-\infty}^{\infty} g(t)\delta(t-t_0)dt = g(t_0)$$

$$\int_{-\infty}^{\infty} g(\tau)\delta(t-\tau)d\tau = g(t)$$

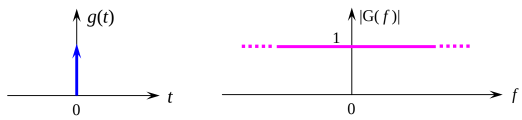
$$g(t) \otimes \delta(t) = g(t)$$

Dirac Delta Fonksiyonu

Temel Bilgiler

- ***Dirac delta fonksiyonunun*** Fourier transformu:

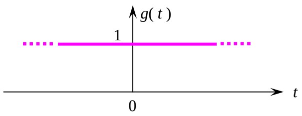
$$F\left[\delta(t)\right] = \int_{-\infty}^{\infty} \delta(t)\, \mathbf{exp}\left(-j2\pi f t\right) dt = 1 \qquad \delta(t) \Leftrightarrow 1$$



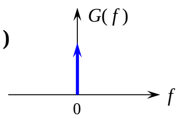
δ(*t*) Uygulamaları

Temel Bilgiler

- ***DC Sinyal*** $1 \Leftrightarrow \delta(f)$

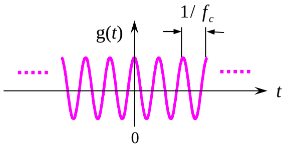


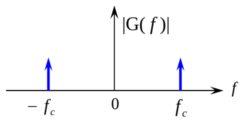
$$\int_{-\infty}^{\infty} \mathbf{exp}\left(-j2\pi f t\right) dt = \delta(f)$$



- ***Kompleks Exponansiyel*** $\exp\left(j2\pi f_c t\right) \Leftrightarrow \delta\left(f - f_c\right)$
- ***Sinüsoidal Sinyal*** $\cos\left(2\pi f_c t\right) \Leftrightarrow \frac{1}{2}\left[\delta\left(f - f_c\right) + \delta\left(f + f_c\right)\right]$

$$\cos\left(2\pi f_c t\right) = \frac{1}{2}\left[\exp\left(j2\pi f_c t\right) + \exp\left(-j2\pi f_c t\right)\right]$$





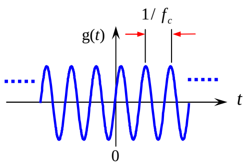
δ(*t*) Uygulamaları

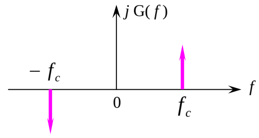
Temel Bilgiler

- ***Sinüsoidal Sinyal***

$$\sin(2\pi f_c t) \Leftrightarrow \frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$$

$$\sin(2\pi f_c t) = \frac{1}{2j}\left[\exp(j2\pi f_c t) - \exp(-j2\pi f_c t)\right]$$





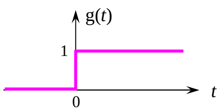
δ(*t*) Uygulamaları

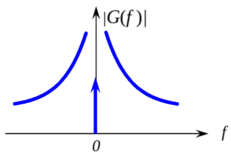
Temel Bilgiler

- ***Birim Basamak Sinyali***

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau \qquad u(t) \Leftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$





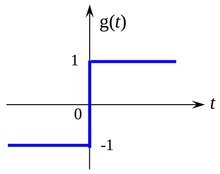
δ(*t*) Uygulamaları

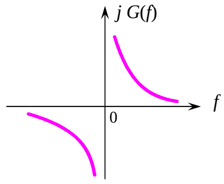
Temel Bilgiler

- ***İşaret Fonksiyonu***

$$\mathrm{sgn}(t) \Leftrightarrow \frac{1}{j\pi f}$$

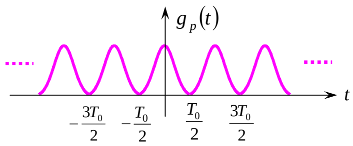
$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1 & t < 0 \end{cases} \qquad \mathrm{sgn}(t) = 2u(t) - 1$$

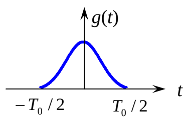




Periyodik Sinyallerin Fourier Transformu

Temel Bilgiler





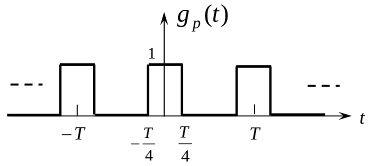
$$g_p(t) = \sum_{m=-\infty}^{\infty} g(t - mT_o)$$

$$g_p(t) \Leftrightarrow \frac{1}{T_o}\sum_{n=-\infty}^{\infty} G\left(\frac{n}{T_o}\right)\delta\left(f - \frac{n}{T_o}\right)$$

Örnek

Temel Bilgiler

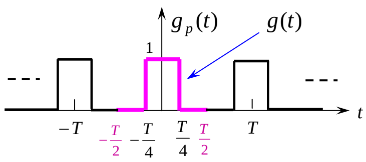
- $g_p(t)$ ***sinyalinin Fourier transformunu bulunuz.***



Çözüm

Temel Bilgiler

- ***g(t) sinyalinin Fourier transformu*** $G(f)=\frac{T}{2}\mathrm{sinc}\left(\frac{fT}{2}\right)$



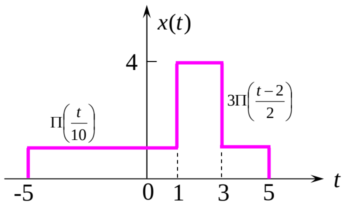
$T_0 = T$

$$g_p(t) \Leftrightarrow \frac{1}{T_o}\sum_{n=-\infty}^{\infty} G\left(\frac{n}{T_o}\right)\delta\left(f-\frac{n}{T_o}\right)$$

$$G_p(f)=\frac{1}{2}\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(\frac{n}{2}\right)\delta\left(f-\frac{n}{T}\right)$$

Örnek

Temel Bilgiler



$$x(t) = 3\Pi\left(\frac{t-2}{2}\right) + \Pi\left(\frac{t}{10}\right)$$

$$X(f) = 6\,\mathrm{sinc}(2f)e^{-j4\pi f} + 10\,\mathrm{sinc}(10f)$$