2014/2 ENGINEERING DEPARTMENTS PHYSICS 2
RECITATION 5
(MAGNETIC FIELDS)

1. A conductor wire carrying a constant current I is in a uniform magnetic field ($\vec{B}$) oriented perpendicularly into the plane of the **Figure 1**. Find the components of the magnetic force on the wire.

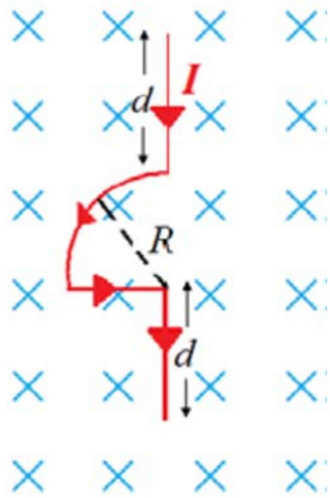
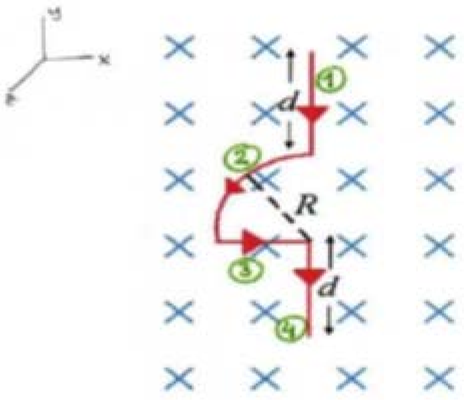


Figure 1



$$\vec{F}_B = I\vec{l} \times \vec{B}$$

for 1. and 4. parts :

$$\vec{F}_1 = \vec{F}_4 = I\vec{d} \times \vec{B}$$

$$\vec{d} = d(-\hat{j})$$

$$\vec{B} = B(-\hat{k})$$

$$\vec{F}_1 = \vec{F}_4 = I d(-\hat{j}) \times B(-\hat{k})$$

$$\boxed{\vec{F}_1 = \vec{F}_4 = I d B \hat{i}}$$

for 3. part :

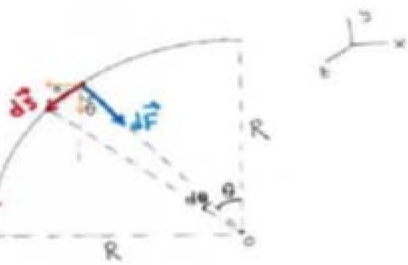
$$\vec{F}_3 = I\vec{R} \times \vec{B}$$

$$\vec{R} = R\hat{i}$$

$$\vec{B} = B(-\hat{k})$$

$$\vec{F}_3 = I R\hat{i} \times B(-\hat{k})$$

$$\boxed{\vec{F}_3 = IRB\hat{j}}$$



for 2. part :

$$d\vec{F}_2 = I\, d\vec{s} \times \vec{B} \qquad ds = R\,d\theta$$

$$d\vec{s} = ds\cos\theta(-\hat{i}) + ds\sin\theta(-\hat{j})$$

$$d\vec{s} = R\cos\theta\, d\theta(-\hat{i}) + R\sin\theta\, d\theta(-\hat{j})$$

$$\vec{F}_2 = I\left(\int_0^{\pi/2} R\cos\theta\, d\theta(-\hat{i}) + \int_0^{\pi/2} R\sin\theta\, d\theta(-\hat{j})\right) \times B(-\hat{k})$$

$$\vec{F}_2 = IR(-\hat{i} - \hat{j}) \times B(-\hat{k})$$

$$\boxed{\vec{F}_2 = IRB(\hat{i} - \hat{j})}$$

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\boxed{\Sigma\vec{F} = IB(2d+R)\hat{i}}$$

2. A closed rectangular loop carrying a constant current I lies in the xy-plane (b is parallel to x-axis and a is parallel to y-axis) as shown in **Figure 2**. The magnetic field is not uniform and given by $\vec{B} = \alpha y \hat{k}$ (α is a constant). Find the components of the net magnetic force on the loop.

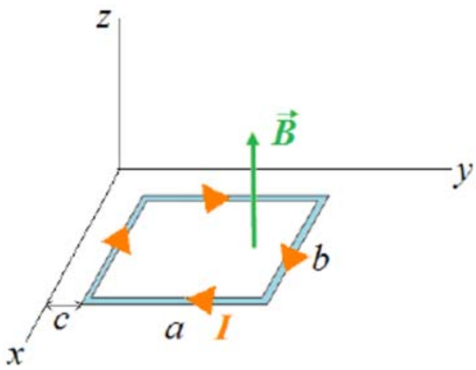
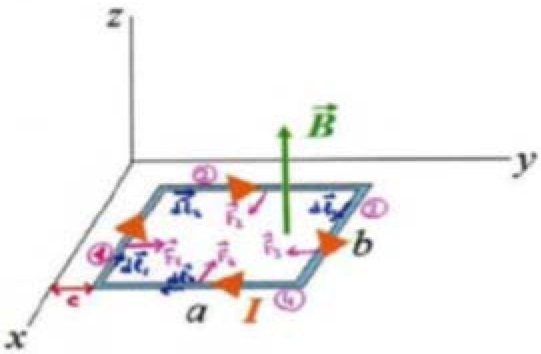


Figure 2

$\vec{F}_B = I\,\vec{\ell} \times \vec{B}$



for 1. part :

$\vec{F}_1 = I \int d\vec{\ell}_1 \times \vec{B}$

$d\vec{\ell}_1 = dx(-\hat{i})$

$\vec{B} = \alpha y \hat{k} = \alpha c \hat{k}$ ($\vec{c}$ cons.)

$\vec{F}_1 = I \int_0^b dx(-\hat{i}) \times (\alpha c \hat{k})$

$\vec{F}_1 = I \alpha c \int_0^b dx\,\hat{j}$; $\boxed{\vec{F}_1 = I \alpha c b\,\hat{j}}$

for 2. part :

$\vec{F}_2 = I \int d\vec{\ell}_2 \times \vec{B}$

$d\vec{\ell}_2 = dy\,\hat{j}$

$\vec{B} = \alpha y \hat{k}$

$$\vec{F}_2 = I \int_c^{c+a} dy\,\hat{j} \times (\alpha y \hat{k}) = I\alpha \int_c^{c+a} y\,dy\,\hat{i} = I\alpha \left[\frac{y^2}{2} \right]_c^{c+a} \hat{i} = I\alpha \left[\frac{(c+a)^2 - c^2}{2} \right] \hat{i}$$

$$\boxed{\vec{F}_2 = I\alpha \left(\frac{a^2 + 2ac}{2} \right) \hat{i}}$$

4. part is same magnitude with 2. part but in the reverse direction

$$\boxed{\vec{F}_4 = I\alpha \left(\frac{a^2 + 2ac}{2} \right) (-\hat{i})}$$

for 3. part :

$\vec{F}_3 = I \int d\vec{\ell}_3 \times \vec{B}$

$d\vec{\ell}_3 = dx\,\hat{i}$

$\vec{B} = \alpha (c+a) \hat{k}$ ($\vec{c}$ cons.)

$\vec{F}_3 = I \int_0^b dx\,\hat{i} \times \alpha (c+a) \hat{k}$; $\boxed{\vec{F}_3 = I\alpha b (c+a)(-\hat{j})}$

$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$\boxed{\Sigma \vec{F} = I\alpha a b\,(-\hat{j})}$

3. A rectangular loop consists of *N=100* closely wrapped turns and has dimensions *a = 1 m* and *b = 2*.The loop is hinged along the *z* axis, and its plane makes an angle *60°* with the *x* axis (**Figure 3**).

(Neglect the magnetic field exerted by the loop)

a) What are the magnitude and direction of the torque on *KL* part of the loop exerted by a uniform magnetic field *B=100 mT* directed along the y axis when the current is in *I=10A* the direction shown?

b) Find the dipole moment of the loop and the torque on the loop exerted by the magnetic field.

c) Calculate the magnetic potential energy of the loop.

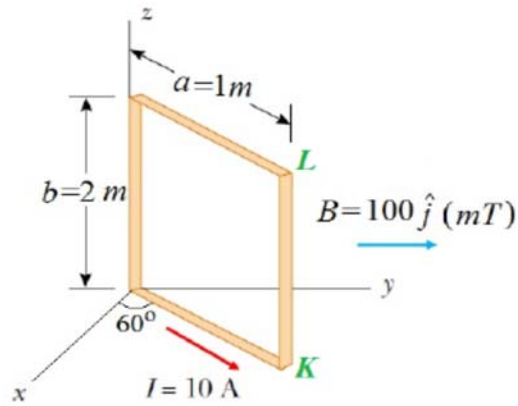


Figure 3

a) $\vec{F}_B = I\vec{\ell} \times \vec{B}$

$B = 100\hat{j}\,(mT) = 0{,}1\hat{j}\,(T)$

N = 100 turns

$\vec{F}_B = I\vec{b} \times \vec{B}$

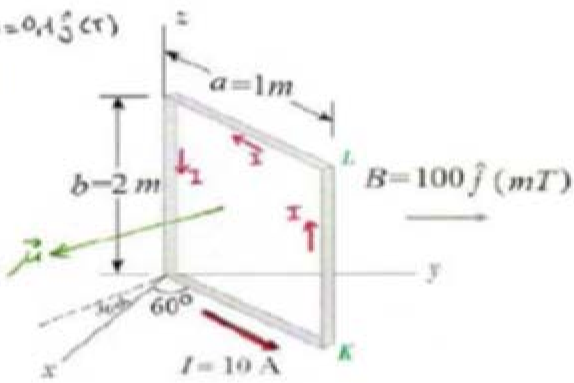
$\vec{b} = 2\hat{k}\,(m)$

$\vec{B} = 0{,}1\hat{j}\,(T)$

$\vec{F}_B = 10.\left[(2\hat{k}) \times (0{,}1\hat{j})\right]$

$\vec{F}_B = -2\hat{i}\,(N)$ (for 1 turn)

for 100 turns : $\boxed{\vec{F}_B = -200\,\hat{i}\,(N)}$



b) $\vec{\mu} = NI\vec{A}$

$A = 2 \times 1 = 2\,(m^2)$

$\mu = 100.10.2 = 2.10^3\,(A.m^2)$

$\vec{\mu} = \mu\cos 30\,\hat{i} - \mu\sin 30°\,\hat{j}$

$\vec{\mu} = 2.10^3\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

$\boxed{\vec{\mu} = 10^3(\sqrt{3}\,\hat{i} - \hat{j})\,(A.m^2)}$

$\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{\tau} = 10^3(\sqrt{3}\,\hat{i} - \hat{j}) \times 0{,}1\hat{j}$

$\boxed{\vec{\tau} = 10^2\sqrt{3}\,\hat{k}\,(N.m)}$

c) $U = -\vec{\mu}.\vec{B}$

$U = -\left[10^3(\sqrt{3}\hat{i} - \hat{j}).0{,}1\hat{j}\right]$

$\boxed{U = 100\,(J)}$

or

$U = -\mu B\cos\theta$

$U = -2.10^3.0{,}1.\cos 120°$

$U = 100\,(J)$

$\mu = \sqrt{3.10^6 + 1.10^6}$

$\mu = 2.10^3\,(A.m^2)$

4. A closed loop carrying a constant current is in a uniform magnetic field given by $\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$ *(T)* (**Figure 4**). Ignoring the magnetic field exerted by the loop, find

a) the magnetic force vector on *MN* part of the loop.

b) the components of the magnetic dipole moment of the loop.

c) the torque acting on the loop and the magnetic potential energy of the loop.

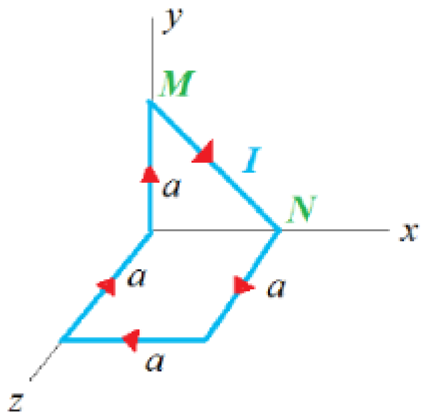
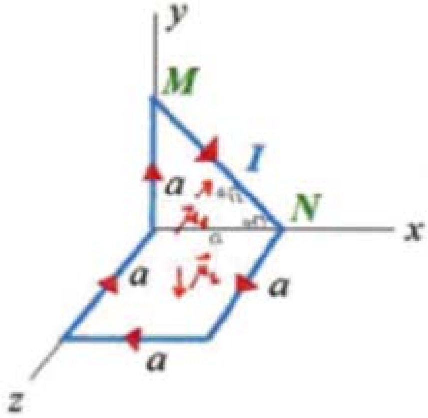


Figure 4



$$\vec{F}_B = I\vec{l} \times \vec{B}$$

a) $\vec{l} = a\sqrt{2}\left(\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right) = a(\hat{i} - \hat{j})$ (m)

$\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$ (T)

$\vec{F}_{MN} = Ia(\hat{i} - \hat{j}) \times (\hat{i} - 2\hat{j} + \hat{k})$

$$\boxed{\vec{F}_{MN} = -Ia(\hat{i} + \hat{j} + \hat{k})\ (N)}$$

b) $\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2$

$\vec{\mu} = I(\vec{A}_1 + \vec{A}_2)$

$\vec{\mu} = I\left[\frac{a^2}{2}(-\hat{k}) + a^2(-\hat{j})\right]$

$$\boxed{\vec{\mu} = -Ia^2\left(\hat{j} + \frac{1}{2}\hat{k}\right)\ (A.m^2)}$$

c) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{\tau} = -Ia^2\left(\hat{j} + \frac{1}{2}\hat{k}\right) \times (\hat{i} - 2\hat{j} + \hat{k})$

$$\boxed{\vec{\tau} = Ia^2\left(-2\hat{i} - \frac{1}{2}\hat{j} + \hat{k}\right)(N.m)}$$

$U = -\vec{\mu} \cdot \vec{B}$

$U = Ia^2\left(\hat{j} + \frac{1}{2}\hat{k}\right) \cdot (\hat{i} - 2\hat{j} + \hat{k})$

$$\boxed{U = -I\frac{3a^2}{2}\ (J)}$$

5. The number of turns of a coil having a section of $6cm^2$ is 50. When the coil is placed in a uniform magnetic field of *0.2T*, the maximum torque is $3.10^{-5}N.m$.

a) Find the magnitude of current on the coil.

b) How much work is done to rotate the coil with the angle of 180^o in the magnetic field?

5) $A = 6\ cm^2 = 6.10^{-4}\ m^2$

$N = 50$ turns

$B = 0{,}2\ T$

$\tau = 3.10^{-5}\ N.m \quad (\vec{\mu} \perp \vec{B})$

a) $\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \tau = \mu B \sin\theta$

$\tau = \mu B \quad (\theta = 90^\circ)$

$3.10^{-5} = \mu . 0{,}2$

$\mu = 1{,}5.10^{-4}\ (A.m^2)$

$\mu = NIA$

$1{,}5.10^{-4} = 50.I.6.10^{-4}$

$I = 5.10^{-3}\ (A)$

$\boxed{I = 5\ (mA)}$

b) $W = \Delta U = U_f - U_i$

$W = (-\vec{\mu}.\vec{B})_f - (-\vec{\mu}.\vec{B})_i$

$W = \mu B(-\cos\theta_f + \cos\theta_i)$

$\theta_i \rightarrow \theta_i + 180^\circ$

$W = \mu B\left[-\cos(\theta_i + 180^\circ) + \cos\theta_i\right]$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$W = 2\mu B \cos\theta_i \quad = 2\tau \cos\theta_i$

$\boxed{W = 6.10^{-5}.\cos\theta_i\ (J)}$

6. A particle having charge q and mass m enters into a velocity selector as to be perpendicular to the electric and the magnetic fields (**Figure 5**). The particle moves with a constant speed in the velocity selector. The particle reaches point P_2 only effect on same magnetic field (B_{in}) from point P_1 by orbital motion. ($B_{in} = 0{,}2\ T$; $E = 4 \times 10^5\ V/m$; $r = 0.1\ m$; $\pi = 3$)

a) Find the velocity of the particle.

b) Determine the direction of the electric field and the sign of the charged particle according to the given coordinate system.

c) Calculate q/m ratio.

d) Find the arrival time of the particle from point P_1 to point P_2.

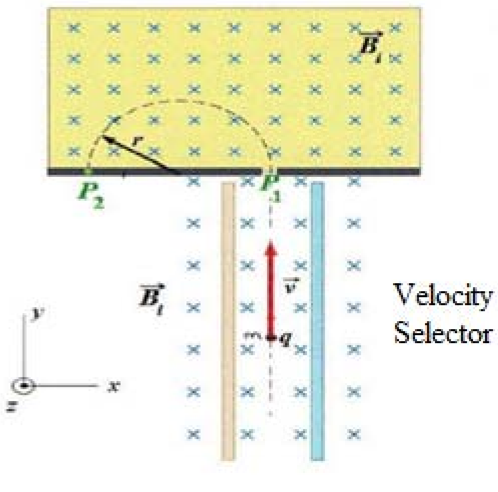
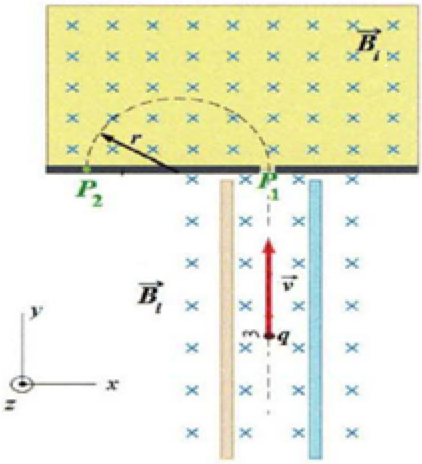


Figure 5

6)



a) Inside of the velocity selector

$\vec{F}_{Net} = 0$

$\vec{F}_{Net} = q\vec{E} + q\vec{v} \times \vec{B} = 0$

$\vec{E} = -\vec{v} \times \vec{B}$

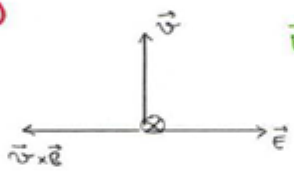
$E = vB\sin 90°$

$v = \dfrac{E}{B}$

$v = \dfrac{4.10^5}{0{,}2}$

$\boxed{v = 2.10^6\ (m/s)}$

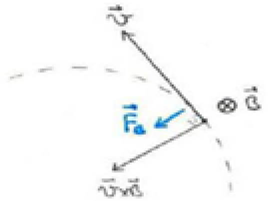
b)



$\vec{E} = -\vec{v} \times \vec{B}$

$\left.\begin{array}{l}\vec{v} = v\hat{j} \\ \vec{B} = B_{in}(-\hat{k})\end{array}\right\}$ $\vec{E} = -\vec{v} \times \vec{B} = -v\hat{j} \times B_{in}(-\hat{k}) = v.B_{in}\,\hat{i}$

in the direction of +x-axis



$\vec{F}_B = q\vec{v} \times \vec{B}$

q is positive charge

c) $qvB = m\dfrac{v^2}{r}$ $(\vec{v} \perp \vec{B})$

$\dfrac{q}{m} = \dfrac{v}{rB}$

$\dfrac{q}{m} = \dfrac{2.10^6}{0{,}1.0{,}2}$

$\boxed{\dfrac{q}{m} = 1.10^8\ (C/kg)}$

d) $t = \dfrac{\pi r}{v}$

$t = \dfrac{3.0{,}1}{2.10^6}$

$\boxed{t = 1{,}5.10^{-7}\ (s)}$

7. A uniform magnetic field of magnitude *0.1 T* is directed along the positive x axis. A positron with a energy of *2 keV enters the field along a direction that makes 85° with the x axis* (**Figure 6**). *The motion of the particle is expected to be a helix.*). Calculate

a) the period of the positron

b) the pitch *p* of helix.

c) the radius *r* of the trajectory.

($m = 9.1 \times 10^{-31}$ kg, $q = 1.6 \times 10^{-19}$ C)

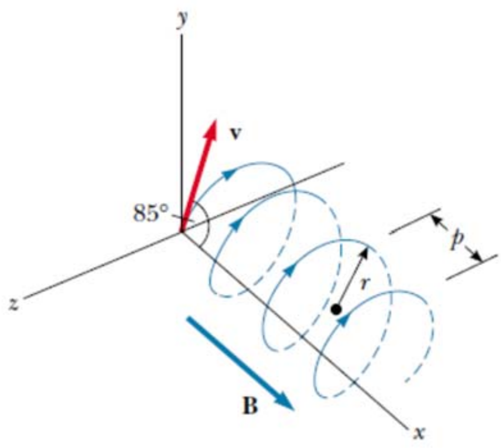


Figure 6

a) $qv_sB = m\frac{v_s^2}{r}$

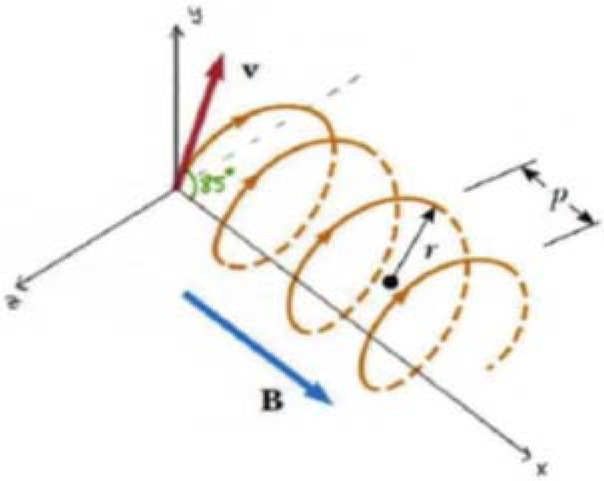
$r = \frac{mv_s}{qB}$

$T = \frac{2\pi r}{v_s}$

$\boxed{T = \frac{2\pi m}{qB}}$

$T = \frac{2\pi\ 9{,}1.10^{-31}}{1{,}6.10^{-19}.\ 0{,}10}$

$\boxed{T = 3{,}57.10^{-10}\ (s)}$



b) The pitch p of trajectory is the distance moved along *x* by the positron during each period

$p = T.v_x$

$p = \frac{2\pi m}{qB}.v\cos 85°$

$p = 3{,}57.10^{-10}.2{,}67.10^{7}.\cos 85°$

$\boxed{p = 8{,}3.10^{-4}\ (m)}$

$K = \frac{1}{2}mv^2$

$2.10^{3}.1{,}6.10^{-19} = \frac{1}{2}.9{,}1.10^{-31}.v^2$

$v = 2{,}65.10^{7}\ (m/s)$

c) $r = \frac{mv_s}{qB}$

$r = \frac{mv\sin 85°}{qB}$

$r = \frac{9{,}1.10^{-31}.\ 2{,}65.10^{7}.\sin 85°}{1{,}6.10^{-19}\ .\ 0{,}10}$

$\boxed{r = 1{,}5.10^{-3}\ (m)}$