# Fuzzy
# Rule Based Systems

Lecture 07

## Natural Language

-Why do we use language ?
  To communicate each other
-What is communication ?
  To transfer information
-What is information ?
  Cognitive picture in one's mind
-Natural language involves a vast amount of information
  " Our language has been termed the shell o our thoughts", Zadeh 1975

Language:
  atoms → molecules → compounds
  words → phrases → sentences

# Natural Language

Fundamental terms: atomic terms
Examples of atomic terms:
"slow" , "medium" , "young" , "beautiful" , etc...
Collection of atomic terms: composites
Examples of composite terms:
"very slow horse" , "medium-weight female" , "young three",
"fairly beautiful painting",etc...
Universe of natural language: X(as element of $\alpha$ ).
Fuzzy set $\underset{\sim}{A}$: universe of interpretations (or meanings)
Mapping from X to Y : $\underset{\sim}{M}(\alpha, \underset{\sim}{A})$

$$\mu_{\underset{\sim}{M}}(\alpha, y) = \mu_{\underset{\sim}{A}}(y)$$

# Natural Language

Example:

$$\underset{\sim}{A} = \text{"}young\text{"} = \int_0^{25} \frac{1}{y} + \int_{25}^{100} \frac{\left(1+\left(\frac{y-25}{5}\right)^2\right)^{-1}}{y}$$

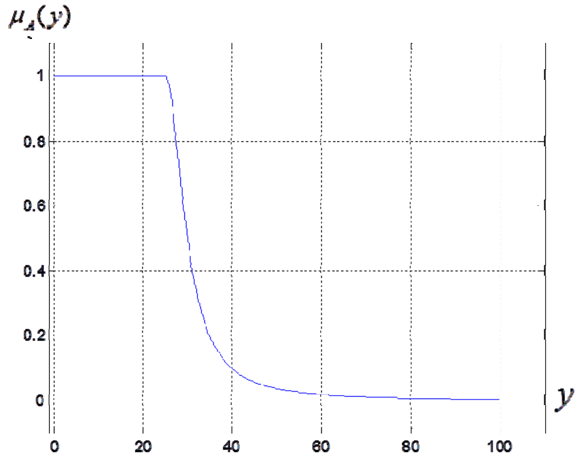
Atomic term: young
Interpolation: $\underset{\sim}{A}$ (interpolation of the term young expressed as a function of age)
Alternative notation:

$$\mu_{\underset{\sim}{A}}(y) = \mu_{\underset{\sim}{M}}(young, y) = , \begin{cases} \left[1+\left(\frac{y-25}{5}\right)^2\right] & , \ y > 25\text{years} \\ 1 & , \ y \leq 25 \text{ years} \end{cases}$$

# Natural Language

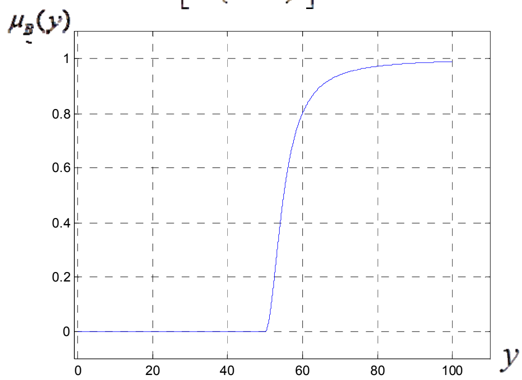
```
y = 0:0.01:100;
output = zeros(length(y),1);
for i = 1:length(y)
    if y(i) <= 25
        output(i,1) = 1;
    else
        output(i,1) = (1+((y(i)-25)/5)^2)^-1;
    end
end
plot(y,output);
grid
axis([-1 110 -0.1 1.1]);
```

# Natural Language



# Natural Language

Example: $\underset{\sim}{B}$ : "old" , $\mu_{\underset{\sim}{B}}(y) = 1 - \left[1 + \left(\frac{y-50}{5}\right)^2\right]^{-1}$ *for* $50 \le y \le 100$




# Natural Language

Example: *Composite*

$\underset{\sim}{A}$ *or* $\underset{\sim}{B}$ : $\mu_{\underset{\sim}{A} \text{ or } \underset{\sim}{B}}(y) = \max\left(\mu_{\underset{\sim}{A}}(y), \mu_{\underset{\sim}{B}}(y)\right)$

("young" or "old")

$\underset{\sim}{A}$ *and* $\underset{\sim}{B}$ : $\mu_{\underset{\sim}{A} \text{ and } \underset{\sim}{B}}(y) = \min\left(\mu_{\underset{\sim}{A}}(y), \mu_{\underset{\sim}{B}}(y)\right)$

("young" and "old")

*Not* $\underset{\sim}{A} = \overline{\underset{\sim}{A}} = \mu_{\overline{\underset{\sim}{A}}}(y) = 1 - \mu_{\underset{\sim}{A}}(y)$

("not young")

# Linguistic Hedges

Fundamental atomic terms are often modified with adjectives or adverbs(verbs).

"very" , "low" , "slight" , "more or less" , "fairly" , "slightly" ," almost", "barely" , "mostly" , "roughly" , "approximately" , etc...

These are called as liguistic hedges.

Define $$\alpha = \int_Y \frac{\mu_A(y)}{y}$$

# Linguistic Hedges

$$\left.\begin{aligned} "very\ \ \alpha" &= \alpha^2 = \int_Y \frac{[\mu_A(y)]^2}{y} \\ "very\ \ very\ \ \alpha" &= \alpha^4 \\ "plus\ \ \alpha" &= \alpha^{1.25} \end{aligned}\right\} concentrations$$

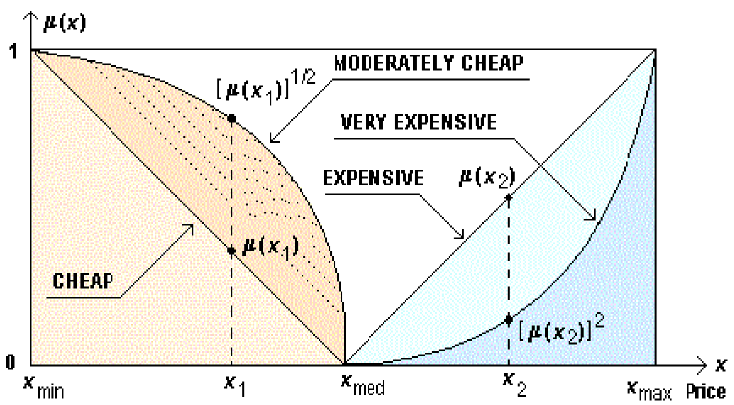
# Linguistic Hedges

$$\text{"}slightly\quad \alpha\text{"} = \sqrt{\alpha} = \int_Y \frac{[\mu_A(y)]^{0.5}}{y}$$

$$\text{"}\min us\quad \alpha\text{"} = \alpha^{0.75}$$

*Dilations*

# Natural Language



# Linguistic Hedges

Consider the domain of attitude control of a spin-stabilized space vehicle. In order to change the attitude of the vehicle, the roll orientation of the vehicle, say $\Phi$ , has to be in a specific position, and the roll rate has to be within a certain bound, say a slow rate and a fast rate. Let these two rates be defined as linguistic variables on a universe of degrees per second:

$$\text{"Fast"} = \left\{ \frac{0}{1} + \frac{0.2}{100} + \frac{0.4}{200} + \frac{0.6}{300} + \frac{0.8}{400} + \frac{1}{500} \right\}$$

$$\text{"Slow"} = \left\{ \frac{1}{1} + \frac{0.8}{100} + \frac{0.6}{200} + \frac{0.4}{300} + \frac{0.2}{400} + \frac{0}{500} \right\}$$

# Linguistic Hedges

$$a)\ \text{"Very Fast"} = (Fast)^2 = \left\{ \frac{0}{1} - \frac{0.04}{100} - \frac{0.16}{200} - \frac{0.36}{300} - \frac{0.64}{400} - \frac{1}{500} \right\}$$

$$b)\ \text{"Fairly Slow"} = (Slow)^{\frac{2}{3}} \left\{ \frac{1}{1} - \frac{0.8618}{100} - \frac{0.7114}{200} - \frac{0.5429}{300} - \frac{0.342}{400} - \frac{0}{500} \right\}$$

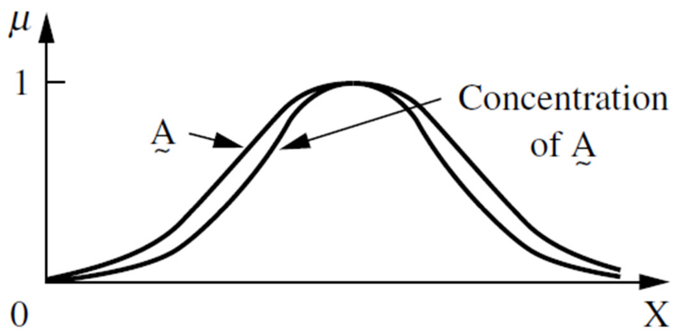
# Linguistic Hedges

Concentrations: Tend to concentrate the elements of a fuzzy set by reducing the degree of memberships.

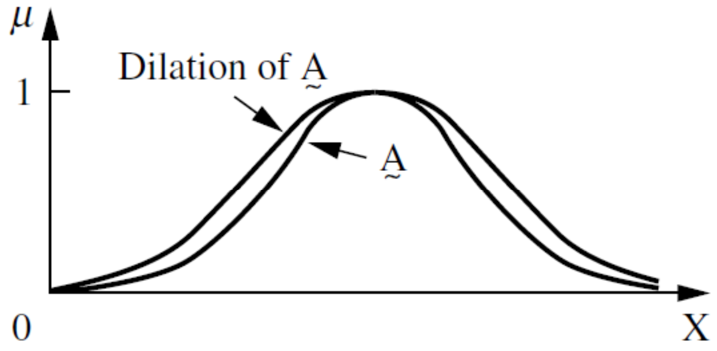
Dilations: Strech or dilate a fuzzy set by increasing membership values.

Intensification: Combination of concentration and dilation.

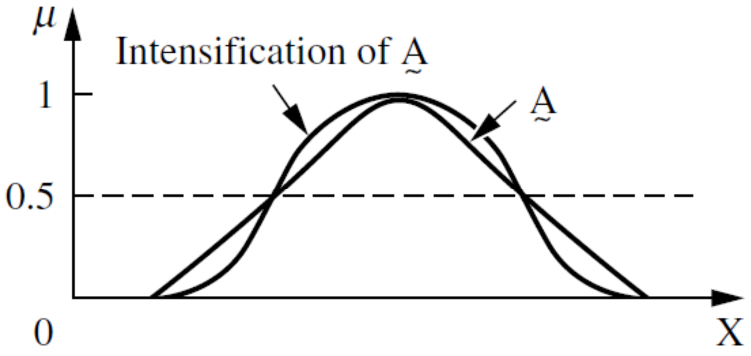
# Linguistic Hedges



# Linguistic Hedges




# Linguistic Hedges



# Linguistic Hedges

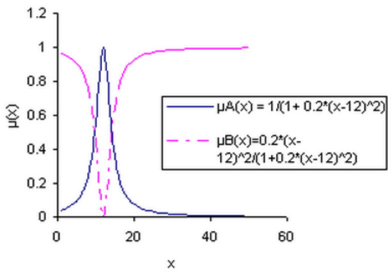
Example: Let us consider two fuzzy sets A and B with membership functions

$$\mu_A(x) = \frac{1}{1+0.2(x-12)^2}$$

$$\mu_B(x) = \frac{0.2(x-12)^2}{1+0.2(x-12)^2}$$

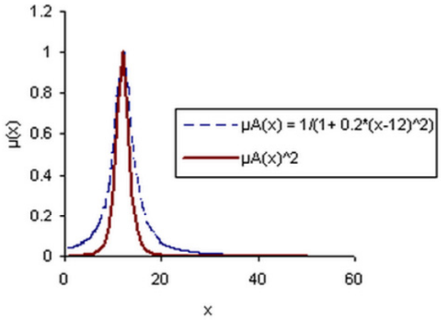
$$= \overline{\mu_A(x)}$$

# Linguistic Hedges



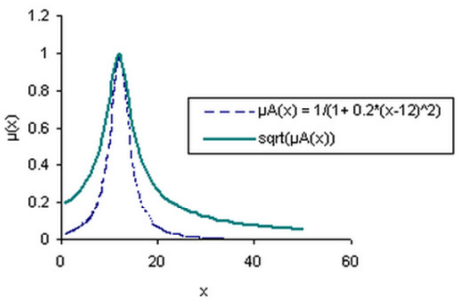

Membership functions $\mu_A(x)$ and $\mu_B(x)$ of Example

# Linguistic Hedges




Concentration of membership function, $\mu_{CON(A)}(x) = [\mu_A(x)]^2$

# Linguistic Hedges




Dilation of membership function, $\mu_{DIL(A)}(x) = \sqrt{\mu_A(x)}$

## Linguistic Hedges

$$\text{"intensify } \alpha\text{"} = \begin{cases} 2\mu_\alpha^{2}(y) & , \quad 0 \le \mu_\alpha(y) \le 0.5 \\ 1 - 2\left[1 - \mu_\alpha(y)\right]^2 & , \quad 0.5 \le \mu_\alpha(y) \le 1 \end{cases}$$

## Fuzzy Rule Based Systems

IF - THEN
RULES

## IF-THEN Rules

- Natural expressions of type:

**“IF premise (antecedent), THEN conclusion (consequent)”**

is called IF-THEN rule-based form.

- Canonical Rule forms:
  - Assignment statements
  - Conditional statements
  - Unconditional statements

## 1.Assignment Statements

- x = large ( x is large)
- Banana’s colour = yellow
- x $\approx$ s
- x is not large and not very small
- Season = winter

## 2.Conditional Statements

- ✓ IF the tomato is red THEN the tomato is ripe.
- ✓ IF the traffic light is red THEN stop.
- ✓ IF x is large THEN y is small ELSE y is not small.
- ✓ *IF* the room is cold THEN turn the heater on.

## 3.Unconditional Statements

- ✓ Go to 9.
- ✓ Divide by x.
- ✓ Turn the temperature higher.
- ✓ Do your homeworks ☺

# IF-THEN Rules

IF condition C' THEN restriction R' ← Canonical rule form

The unconditional restictions might be in the form;

$R^1$ : *The output is* $B^1$

*AND*

$R^2$ : *The output is* $B^2$

*AND*

*etc...*

# Multiple Conjunctive Antecendents

$$IF\ \ x\ is\ \underset{\sim}{A}^1\ \ and\ x\ is\ \underset{\sim}{A}^2\ \ldots\ and\ \ x\ is\ \underset{\sim}{A}^L\ THEN\ \ y\ is\ \underset{\sim}{B}^S$$

$$\mu_{\underset{\sim}{A}^s}(x) = \min\left[\mu_{\underset{\sim}{A}^1}(x), \mu_{\underset{\sim}{A}^2}(x), \ldots, \mu_{\underset{\sim}{A}^L}(x)\right]$$

$$\Downarrow$$

$$IF\ \ \underset{\sim}{A}^s\ THEN\ \underset{\sim}{B}^s$$

# Multiple Disjunctive Antecendents

$$IF \;\; x \text{ is } \underset{\sim}{A}^1 \;\; \text{or} \;\; x \text{ is } \underset{\sim}{A}^2 \ldots \;\; \text{or} \;\; x \text{ is } \underset{\sim}{A}^L \; THEN \; y \text{ is } \underset{\sim}{B}^s$$

$$\underset{\sim}{A}^s = \underset{\sim}{A}^1 \cup \underset{\sim}{A}^2 \cup \ldots \cup \underset{\sim}{A}^L$$

$$\Downarrow$$

$$IF \;\; \underset{\sim}{A}^s \;\; THEN \;\; \underset{\sim}{B}^S$$

# Conditional Statements with ELSE and UNLESS

a) $IF \;\; \underset{\sim}{A}^1 \;\; THEN \left( \underset{\sim}{B}^1 \;\; ELSE \;\; \underset{\sim}{B}^2 \right)$

$$Decomposition: \;\; IF \;\; \underset{\sim}{A}^1 \;\; THEN \;\; \underset{\sim}{B}^1$$

$$OR$$

$$IF \;\; NOT \;\; \underset{\sim}{A}^1 \;\; THEN \;\; \underset{\sim}{B}^2$$

## Conditional Statements with ELSE and UNLESS

b) $IF \ \underset{\sim}{A}^1 \left( THEN \ \underset{\sim}{B}^1 \right) UNLESS \ \underset{\sim}{A}^2$

$$Decomposition: \quad IF \ \underset{\sim}{A}^1 \ THEN \ \underset{\sim}{B}^1$$

$$OR$$

$$IF \ \underset{\sim}{A}^2 \ THEN \ NOT \ \underset{\sim}{B}^1$$

## Conditional Statements with ELSE and UNLESS

c) $IF \ \underset{\sim}{A}^1 \ THEN \left( \underset{\sim}{B}^1 \ ELSE \ IF \ \underset{\sim}{A}^2 \ THEN \left( \underset{\sim}{B}^2 \right) \right)$

$$Decomposition: \quad IF \ \underset{\sim}{A}^1 \ THEN \ \underset{\sim}{B}^1$$

$$OR$$

$$IF \ NOT \ \underset{\sim}{A}^1 \ AND \ \underset{\sim}{A}^2 \ THEN \ \underset{\sim}{B}^2$$

# Nested IF-THEN Rules

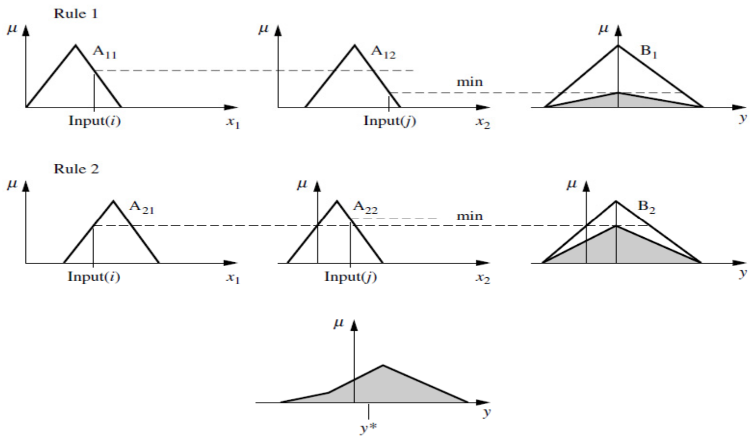
$$IF\ \underset{\sim}{A}^1\ THEN\ \left(IF\ \underset{\sim}{A}^2\ THEN\left(\underset{\sim}{B}^1\right)\right)$$

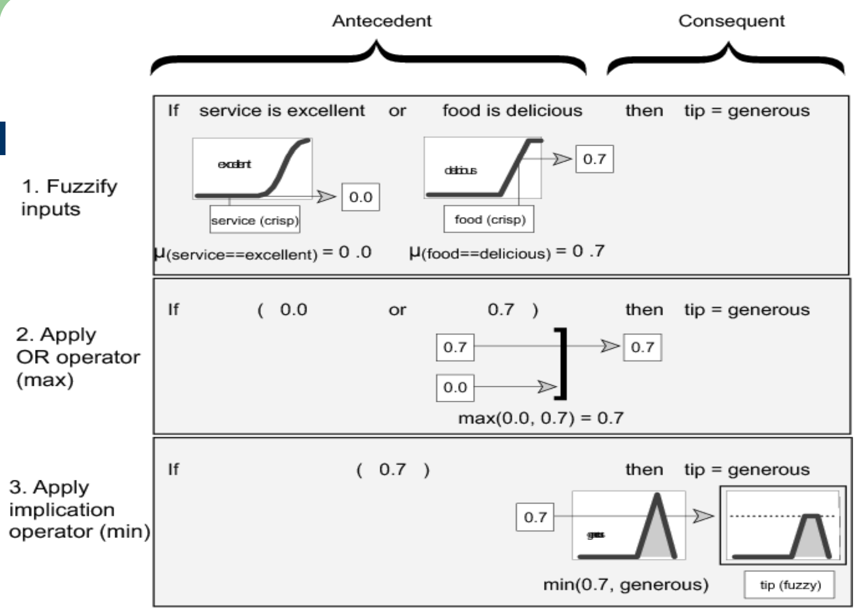
$$Decomposition:\ IF\ \underset{\sim}{A}^1\ AND\ \underset{\sim}{A}^2\ THEN\ \underset{\sim}{B}^1.$$

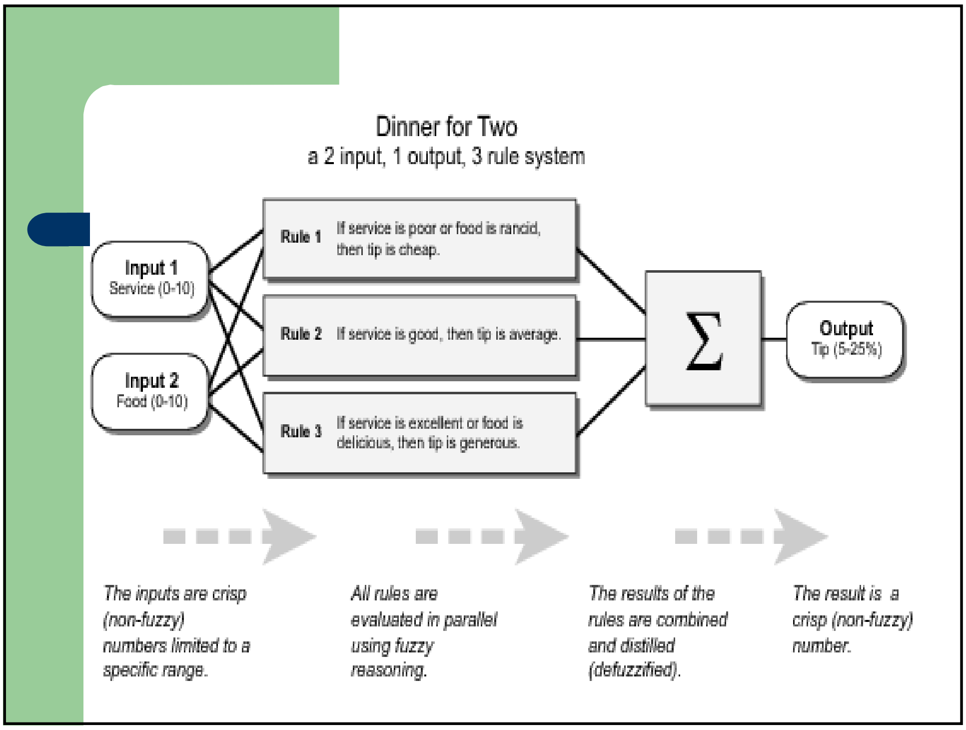
**Example** In mechanics, the energy of a moving body is called kinetic energy. If an object of mass $m$ (kilograms) is moving with a velocity $v$ (meters per second), then the kinetic energy $k$ (in joules) is given by the equation $k = \frac{1}{2}mv^2$. Suppose we model the mass and velocity as inputs to a system (moving body) and the energy as output, then observe the system for a while and deduce the following two disjunctive rules of inference based on our observations:

Rule 1 : IF $x_1$ is $\underset{\sim}{A}_1^1$ (small mass) *and* $x_2$ is $\underset{\sim}{A}_2^1$ (high velocity),
THEN $y$ is $\underset{\sim}{B}^1$ (medium energy).

Rule 2 : IF $x_1$ is $\underset{\sim}{A}_1^2$ (large mass) *or* $x_2$ is $\underset{\sim}{A}_2^2$ (medium velocity),
THEN $y$ is $\underset{\sim}{B}^2$ (high energy).


Rule 1
μ
A11
Input(i)
x1
A12
min
Input(j)
x2
B1
y
Rule 2
A21
Input(i)
A22
min
Input(j)
B2
y*


Antecedent
Consequent
If service is excellent or food is delicious then tip = generous
1. Fuzzify inputs
service (crisp)
0.0
food (crisp)
0.7
μ(service==excellent) = 0 .0
μ(food==delicious) = 0 .7
2. Apply OR operator (max)
If ( 0.0 or 0.7 ) then tip = generous
0.7
0.0
0.7
max(0.0, 0.7) = 0.7
3. Apply implication operator (min)
If ( 0.7 ) then tip = generous
0.7
min(0.7, generous)
tip (fuzzy)


Dinner for Two
a 2 input, 1 output, 3 rule system
Input 1
Service (0-10)
Input 2
Food (0-10)
Rule 1
If service is poor or food is rancid, then tip is cheap.
Rule 2
If service is good, then tip is average.
Rule 3
If service is excellent or food is delicious, then tip is generous.
Σ
Output
Tip (5-25%)
The inputs are crisp (non-fuzzy) numbers limited to a specific range.
All rules are evaluated in parallel using fuzzy reasoning.
The results of the rules are combined and distilled (defuzzified).
The result is a crisp (non-fuzzy) number.